

A satellite view of the Earth showing the Gulf of Mexico and surrounding landmasses, positioned in the top left corner of the slide.

Adaptive Flight Control of Missiles: Needs and Challenges

A satellite view of the Earth showing the Gulf of Mexico and surrounding landmasses, positioned in the bottom left corner of the slide.

**Workshop on Guidance, Navigation
and Control Applications in the
Aerospace Industry**

Dr. James Fisher

Dr. D. Brett Ridgely

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Raytheon Missile Systems Products

- One of the premier developers and manufacturers of tactical missile systems in the world – Headquarters in Tucson, AZ



<http://www.raytheon.com/capabilities/precision/>

Overview and Objective of this Talk

- Modern missile systems must deliver very high performance and reliability at the lowest possible cost
 - Pushes systems into nonlinear regions, uncertainties grow
 - These systems have no reuse, so they cannot have the highest quality nor redundant subsystems
 - Desire reduced design cycle costs (less pre-flight/pre-production testing)
 - Fortunately, cost of processors is decreasing and throughput is increasing
- Adaptive flight controllers may provide a good solution to this challenge
 - There are several practical design challenges for adaptive controllers for high performance missile systems – these will be discussed in this talk
 - Do we really need an adaptive controller? Can we just design a more robust non-adaptive controller? We will examine this here
- There are numerous adaptive approaches in the literature
 - Two of the major adaptive “camps” are represented at this workshop – MRAC and L1 – details left to these speakers, brief summary of each on next slide
 - Another emerging method is RCAC – a quick summary is found on Slide 5
 - Discussion here centers around direct rather than indirect adaptive control due to required speed of airframe response, which drives overall system performance
 - Indirect Adaptive – Plant parameters estimated on-line, controller parameters are adjusted based on estimates
 - Direct Adaptive – No effort made to ID plant parameters, control parameters directly adjusted to improve performance

This talk is NOT about any particular adaptive method!

Model Reference Adaptive Control (MRAC) Overview

Robust and Adaptive Control with Aerospace Applications, Eugene Lavretsky and Kevin Wise, Springer 2013
and another talk in this workshop

- Open loop system Dynamics

$$\dot{x} = Ax + B\Lambda(u + \theta^T \Phi(x)) + B_{ref} z_{cmd}$$

$$y = Cx \quad z = C_2 x$$

- Baseline output feedback control law

$$u_N = -K_{AP} y$$

- Closed loop reference model

$$\dot{\hat{x}} = A_{ref} \hat{x} + B_{ref} z_{cmd} + L(y - \hat{y})$$

$$\hat{y} = C_{ref} \hat{x}$$

- Adaptive learning law

$$\dot{\hat{\theta}} = -\text{Proj}[\bar{\theta}, \Gamma(e^T PB\Phi(x) - \lambda|e^T PB\Phi(x)|\hat{\theta})]$$

- Adaptive control law

$$u = u_N + u_{ad} = -K_{AP} y - \hat{\theta}^T \Phi(x)$$

L1 Adaptive Control Overview

L1 Adaptive Control Theory: Guaranteed Robustness with Fast Adaptation, Naira Hovakimyan and Chengyu Cao, SIAM 2010
and another talk in this workshop

- Open loop system dynamics

$$\dot{x} = Ax + B(\theta^T x + u)$$

$$y = Cx$$

- State predictor

$$\dot{\hat{x}} = A\hat{x} + B(\hat{\theta}^T x + \hat{u})$$

$$\hat{y} = C\hat{x}$$

- Control laws

$$u = -Ky + u_{ad}$$

$$\hat{u} = -K\hat{y} + u_{ad}$$

- Adaptive learning law

$$\dot{\hat{\theta}} = \Gamma(\hat{x} - x)PBx$$

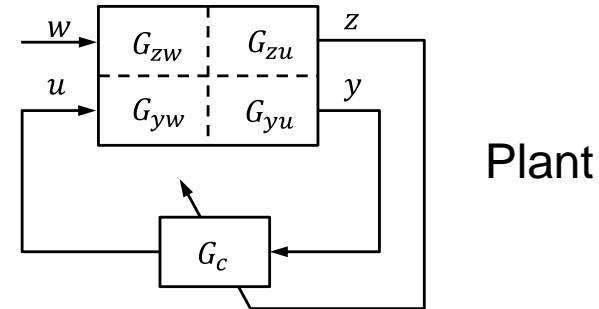
- Adaptive control law

$$u_{ad} = C_{LP}(s)\hat{\theta}^T x$$

Retrospective Cost Adaptive Control (RCAC) Overview

- RCAC is a discrete-time direct adaptive control algorithm
 - Considers actual control actions over a trailing window
 - Basic idea is to re-optimize the control since past control is known as well as the consequences of using said control
- Unlike MRAC or L1 adaptive control, RCAC does not use a Lyapunov-based learning law, but utilizes a gradient-based optimization

Adaptive Control Based on Retrospective Cost Optimization, Mario Santillo and Dennis Bernstein, Journal of Guidance, Control, and Dynamics. Vol 33, No 2, 2010



- Controller construction

$$u(k) = \phi(k)\theta(k)$$

$$\theta(k)^T = [M_1(k) \quad \dots \quad M_{n_c}(k) \quad N_1(k) \quad \dots \quad N_{n_c}(k)]$$

Adaptive Gain Matrix

$$\phi(k) = [u(k-1) \quad \dots \quad u(k-n_c) \quad y'(k-1) \quad \dots \quad y'(k-n_c)]$$

Regressor Matrix

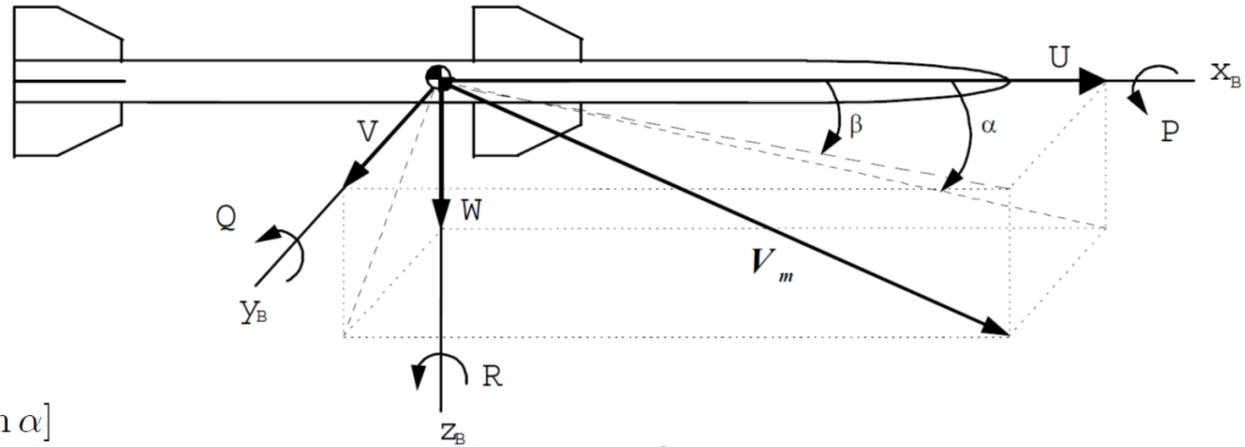
- Construction of the retrospective performance variable

$$\hat{z}(k) = z(k) - G_f(\mathbf{z})[u(k) - \phi(k)\hat{\theta}(k)]$$

- Construction and solution of the retrospective cost function

$$J(k, \hat{\theta}) = \sum_{i=k_0}^k \hat{z}(i)^T R_z \hat{z}(i) + [\hat{\theta} - \theta(k_0)]^T R_\theta [\hat{\theta} - \theta(k_0)]$$

Nonlinear Equations of Motion for a Missile



Translational

$$\dot{V}_m = \frac{1}{\bar{V}} [A_X + A_Y \tan \beta + A_Z \tan \alpha]$$

$$\dot{\alpha} = -P \cos^2 \alpha \tan \beta + Q - R \sin \alpha \cos \alpha \tan \beta - A_X \frac{\bar{V} \sin \alpha \cos \alpha}{V_m} + A_Z \frac{\bar{V} \cos^2 \alpha}{V_m}$$

$$\dot{\beta} = P \tan \alpha \cos^2 \beta + Q \tan \alpha \sin \beta \cos \beta - R - A_X \frac{\bar{V} \sin \beta \cos \beta}{V_m} + A_Y \frac{\bar{V} \cos^2 \beta}{V_m}$$

Rotational

$$\dot{P} = \frac{1}{(I_{xx}I_{zz} - I_{xz}^2)} \left[\begin{aligned} & [I_{xz} (I_{xx} + I_{zz} - I_{yy})] PQ + [I_{zz} (I_{yy} - I_{zz}) - I_{xz}^2] QR \\ & + [I_{zz}\mathcal{L} + I_{xz}\mathcal{N}] + I_{zz}M_{T_X} + I_{xz}M_{T_Z} \end{aligned} \right]$$

$$\dot{Q} = \frac{1}{I_{yy}} [I_{xz} (R^2 - P^2) + (I_{zz} - I_{xx}) PR + \mathcal{M} + M_{T_Y}]$$

$$\dot{R} = \frac{1}{(I_{xx}I_{zz} - I_{xz}^2)} \left[\begin{aligned} & [I_{xx} (I_{xx} - I_{yy}) + I_{xz}^2] PQ + [I_{xz} (I_{yy} - I_{xx} - I_{zz})] QR \\ & + [I_{xz}\mathcal{L} + I_{xx}\mathcal{N}] + I_{xz}M_{T_x} + I_{xx}M_{T_z} \end{aligned} \right]$$

IMU Accelerations

$$A_{X_{IMU}} = \bar{A}_X - (Q^2 + R^2) \bar{x} + (PQ - \dot{R}) \bar{y} + (PR + \dot{Q}) \bar{z}$$

$$A_{Y_{IMU}} = \bar{A}_Y + (PQ + \dot{R}) \bar{x} - (P^2 + R^2) \bar{y} + (QR - \dot{P}) \bar{z}$$

$$A_{Z_{IMU}} = \bar{A}_Z + (PR - \dot{Q}) \bar{x} + (QR + \dot{P}) \bar{y} - (P^2 + Q^2) \bar{z}$$

where

$$A_X = \frac{F_X}{m} + \frac{T_X}{m} - g \sin \Theta$$

$$A_Y = \frac{F_Y}{m} + \frac{T_Y}{m} + g \sin \Phi \cos \Theta$$

$$A_Z = \frac{F_Z}{m} + \frac{T_Z}{m} + g \cos \Phi \cos \Theta$$

$$\bar{V} \equiv \sqrt{1 + \tan^2 \alpha + \tan^2 \beta}$$

$$\bar{A}_X = \frac{F_X}{m} + \frac{T_X}{m}$$

$$\bar{A}_Y = \frac{F_Y}{m} + \frac{T_Y}{m}$$

$$\bar{A}_Z = \frac{F_Z}{m} + \frac{T_Z}{m}$$

Linear Decoupled Pitch Plane EOM

➤ To get to this level of simplification, must assume:

- Thrust on centerline
- No thrust vectoring or reaction jets
- Forces and moments are linear wrt states
- Changes in altitude about trim are small
- Trim roll rate is zero
- Thrust not throttle-able
- Change in velocity about trim is small
- Ignore gravity
- Missile is cruciform
- IMU on the centerline
- Three axes are decoupled

$$\begin{aligned}\dot{x}_p &= \hat{A}_p x_p + \hat{B}_p u_p \\ y_p &= \hat{C}_p x_p + \hat{D}_p u_p\end{aligned}$$

where

$$x_p = \begin{bmatrix} \delta\alpha \\ \delta q \end{bmatrix} \quad u_p = [\delta(\delta p)] \quad y_p = \begin{bmatrix} \delta q \\ \delta\alpha_{z_{IMU}} \end{bmatrix}$$

and

$$\hat{A}_p = \begin{bmatrix} \hat{a}_{11} & 1 \\ \hat{a}_{41} & \hat{a}_{44} \end{bmatrix} \quad \hat{B}_p = \begin{bmatrix} \hat{b}_{12} \\ \hat{b}_{42} \end{bmatrix} \quad \hat{C}_p = \begin{bmatrix} 0 & 1 \\ \hat{c}_{31} & \hat{c}_{34} \end{bmatrix} \quad \hat{D}_p = \begin{bmatrix} 0 \\ \hat{d}_{32} \end{bmatrix}$$

where

$$\hat{a}_{11} = \begin{bmatrix} \frac{\tan \alpha_0}{\bar{V}_0 V_{m0}} (A_{Z_0} - A_{X_0} \tan \alpha_0) - R_0 (\cos^2 \alpha_0 - \sin^2 \alpha_0) \tan \beta_0 \\ -\frac{\bar{V}_0 \cos \alpha_0}{m V_{m0}} (X_{\alpha_0} \sin \alpha_0 - Z_{\alpha_0} \cos \alpha_0) \\ -\frac{\bar{V}_0}{V_{m0}} \{ A_{X_0} (\cos^2 \alpha_0 - \sin^2 \alpha_0) + 2A_{Z_0} \sin \alpha_0 \cos \alpha_0 \} \end{bmatrix}$$

$$\hat{a}_{41} = \frac{M_{\alpha_0}}{I_{yy}} \quad \hat{a}_{44} = \frac{M_{Q_0}}{I_{yy}}$$

$$\hat{b}_{12} = -\frac{\bar{V}_0 \cos \alpha_0}{m V_{m0}} [X_{\delta p_0} \sin \alpha_0 - Z_{\delta p_0} \cos \alpha_0] \quad \hat{b}_{42} = \frac{M_{\delta p_0}}{I_{yy}}$$

$$\hat{c}_{31} = \frac{Z_{\alpha_0}}{m} - \frac{M_{\alpha_0}}{I_{yy}} \bar{x}$$

$$\hat{d}_{32} = \frac{Z_{\delta p_0}}{m} - \frac{M_{\delta p_0}}{I_{yy}} \bar{x}$$

$$\hat{c}_{34} = -\frac{M_{Q_0}}{I_{yy}} \bar{x}$$

Adaptive Control of Missiles

- Numerous challenges to the application of adaptive control to missiles will be discussed in the following slides
 - Unmatched uncertainty
 - Non-minimum phase zero dynamics
 - Large flight envelope
 - Quickly varying nonlinear aerodynamics
 - Significant coupling between control channels
 - Actuator dynamics
- Several benefits of adaptive control will then be discussed, followed by simulation and flight results

Uncertainty Modeling

- Consider the simple pitch plane dynamics

$$\begin{aligned} \begin{bmatrix} \dot{\alpha} \\ \dot{q} \end{bmatrix} &= \begin{bmatrix} \frac{Z_\alpha}{mV} & 1 \\ \frac{M_\alpha}{I_{yy}} & \frac{M_q}{I_{yy}} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} \frac{Z_\delta}{mV} \\ \frac{M_\delta}{I_{yy}} \end{bmatrix} \delta \\ &= \begin{bmatrix} \frac{\rho V S C_{z\alpha}}{2m} & 1 \\ \frac{\rho V^2 S c C_{m\alpha}}{2I_{yy}} & \frac{\rho V^2 S c C_{mq}}{2I_{yy}} \end{bmatrix} \begin{bmatrix} \alpha \\ q \end{bmatrix} + \begin{bmatrix} \frac{\rho V S C_{z\delta}}{2m} \\ \frac{\rho V^2 S c C_{m\delta}}{2I_{yy}} \end{bmatrix} \delta \end{aligned}$$

- Different types of uncertainty must be accounted for in the flight control system design

- $V = V_{nom} + \Delta V$: Missile velocity is usually based on navigation estimates which is usually a ground relative velocity
- $\rho = \rho_{nom} + \Delta\rho$: Atmospheric density is not usually measured and the controller may have an implicit day-type assumption

- $m = m_{nom}(1 + \Delta m)$, $I_{yy} = I_{yy,nom}(1 + \Delta I_{yy})$: System mass and inertia are usually very well characterized, but variation may occur in production as well as in burn state for systems with no mass flow measurement
- $C_{z\alpha} = C_{z\alpha}(1 + \Delta C_{z\alpha})$, $C_{z\delta} = C_{z\delta}(1 + \Delta C_{z\delta})$: Force coefficients are characterized in the wind tunnel or using CFD. These coefficients may have errors due to uncertainty in measurements, model build-up methodology, configuration changes, and vehicle deformation
- $C_{m\alpha} = C_{m\alpha}(1 + \Delta C_{m\alpha}) \approx C_{z\alpha}(1 + \Delta C_{z\alpha})(x_{cp} + \Delta x_{cp})$: Stability coefficient can be affected by the body force coefficient as well as by a center of pressure shift
- $C_{m\delta} = C_{m\delta}(1 + \Delta C_{m\delta}) \approx C_{z\delta}(1 + \Delta C_{z\delta})(x_{fin} + \Delta x_{fin})$: Control moment can be affected by control power or effective force application point

- All of these types of uncertainty are at least partially unmatched!

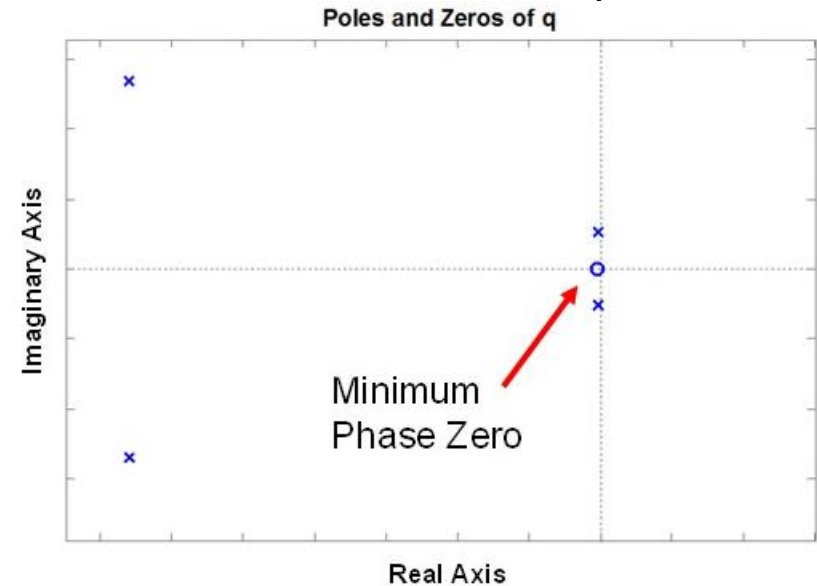
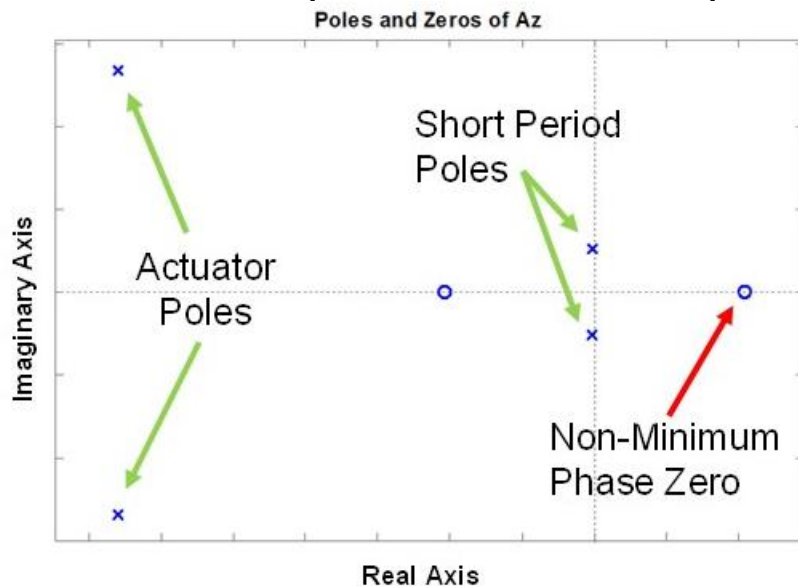
$$\Delta A \neq B\theta^T$$

Output Feedback

- Typically, full state feedback is not practical for missiles
 - Packaging (size, weight and power) and cost constraints typically prevent the use of air data systems for most missiles
 - Subsonic systems (such as UAS platforms) may have an air data system, but this is not common in most lower-speed missiles
- Acceleration feedback is a natural choice of control variable
 - Missile guidance loops typically command accelerations which are generated from line-of-sight to target information
 - IMU used to measure body accelerations and angular rates
 - For tail controlled systems – acceleration output is non-minimum phase
- Methods to apply adaptive control to non-minimum phase systems generally require modifying the measurement
 - Synthetically shifting the IMU to create a minimum phase output
 - Using rate measurements alone as the output variable, or trying to define other controlled variables
 - Use of a state estimator to obtain unmeasured states for state feedback

Dealing with Non-Minimum Phase Zeroes

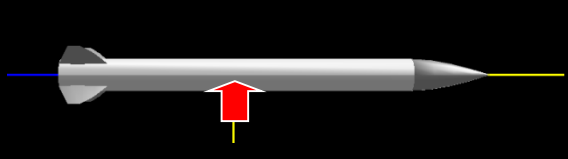
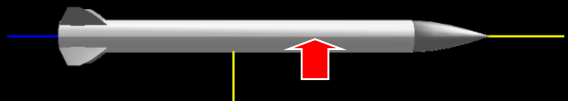

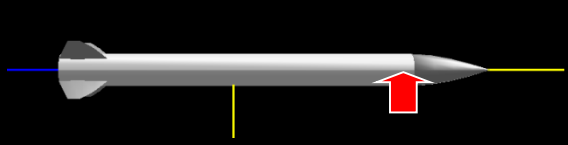
- Acceleration measured at or near the CG of a tail controlled missile is non-minimum phase
 - To accelerate upwards, we must pitch the nose up (positive pitch moment) to generate positive angle of attack
 - When the tails are behind the CG, this requires generation of an downward force to turn the vehicle
 - The body initially accelerates downward until enough angle-of-attack is generated to produce an upward lift from the body
 - This “wrong way initial response” is typical in a system with zeroes in the right half plane
- Pitch rate response is minimum phase and can also be used as an output



Virtual IMU Acceleration

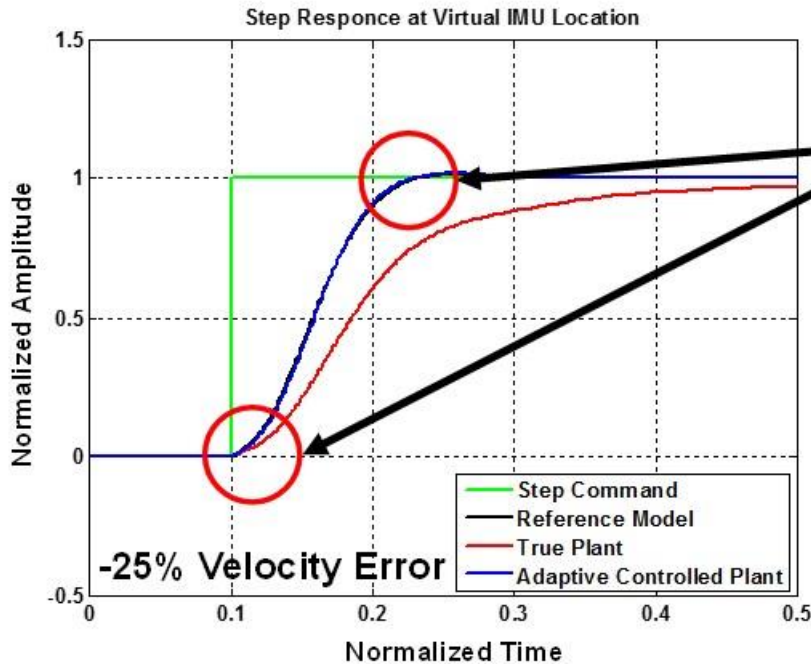
- One standard method of dealing with the non-minimum phase problem of tail-controlled vehicles is to control the acceleration not at the actual IMU location, but at a virtual IMU location. If the virtual IMU location is forward of the instantaneous center of rotation (ICR), the measured acceleration will be minimum phase

Generic Missile Example

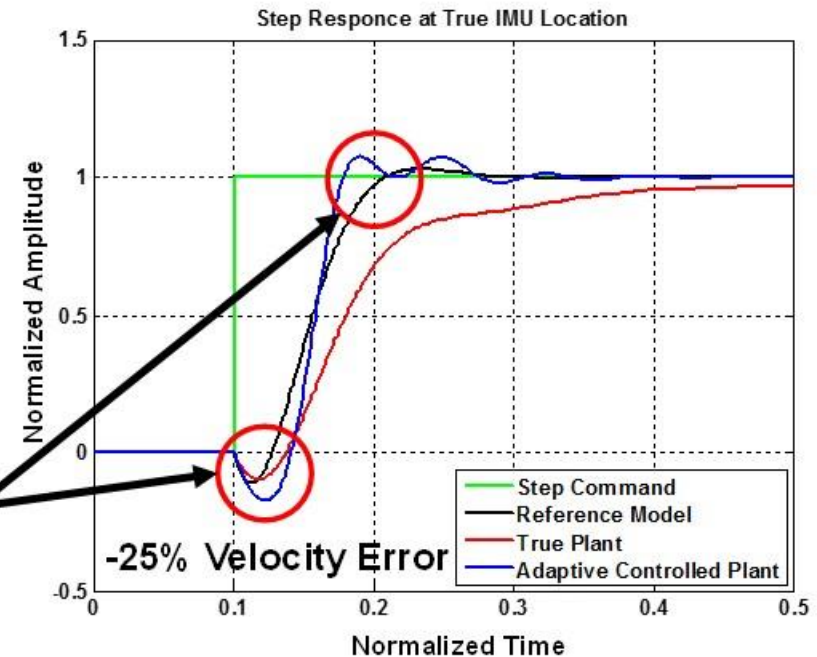
$\bar{A}_{z_m}^M = A_{z_m}^M - \bar{x} \dot{q}_m^M$	A_z measured at:	$G_{A_z, \delta}(s) =$
	Center of Gravity • Non-minimum phase	$\frac{-0.20 (s-26.17) (s+26.17)}{(s+6.00) (s-5.40)}$
	IMU • Non-minimum phase	$\frac{-0.12 (s-34.53) (s+34.08)}{(s+6.00) (s-5.40)}$
	Center of Rotation (ICR) • Zero goes to $+\infty$ • Flight condition dependent	$\frac{0.13 (s+1082.8)}{(s+6.00) (s-5.40)}$
	Forward of ICR • Minimum phase • Zero returns from $-\infty$ • Could even be fore of nose!	$\frac{0.009 (s+7.46+124.17i) (s+7.46-124.17i)}{(s+6.00) (s-5.40)}$

Performance when Adapting to Virtual IMU Acceleration Output

If uncertainty is **matched** (i.e. control effectiveness), then we get **excellent** performance at the true IMU location; however, if uncertainty is unmatched ...



- At the virtual IMU location, the acceleration is **minimum** phase
- The adaptive plant's virtual IMU tracks the reference model's virtual IMU

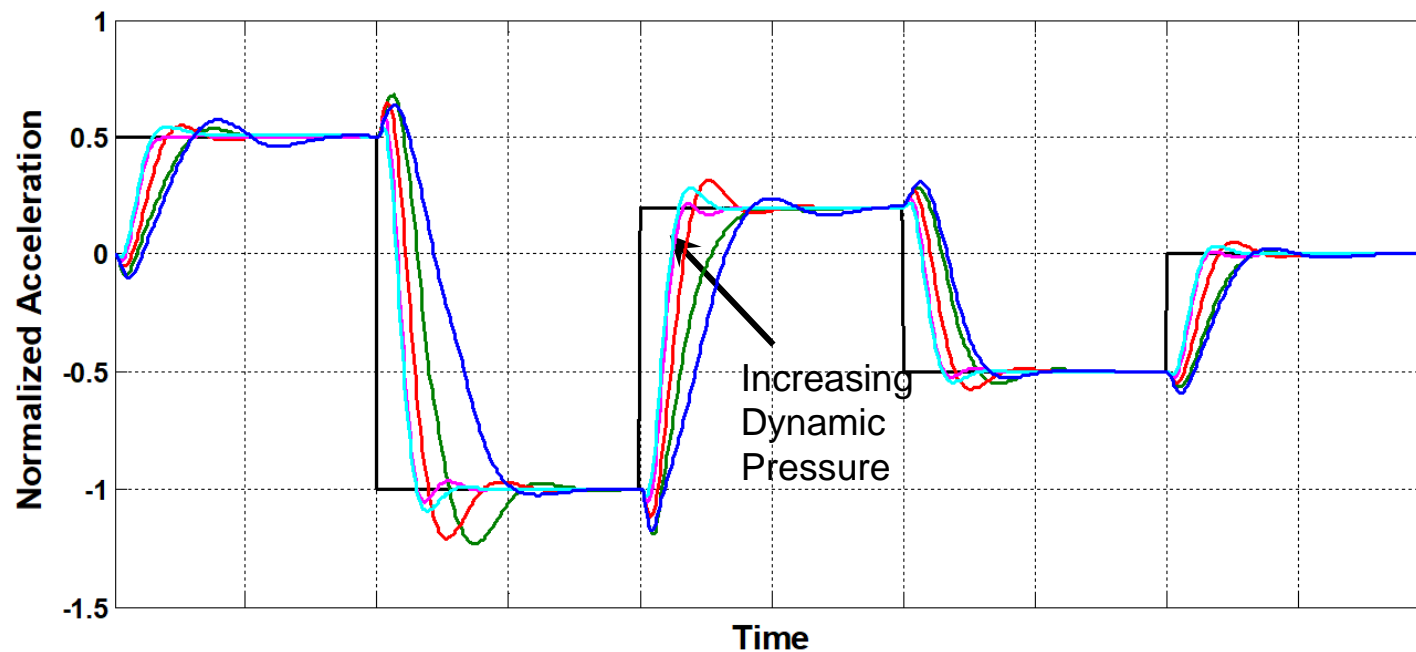


- At the true IMU location, the acceleration is still **non-minimum** phase
- The adaptively controlled plant is now **stable** but shows **highly oscillatory** behavior (poor damping)

Unmatched Uncertainty can Result in Oscillations in the True IMU Accel

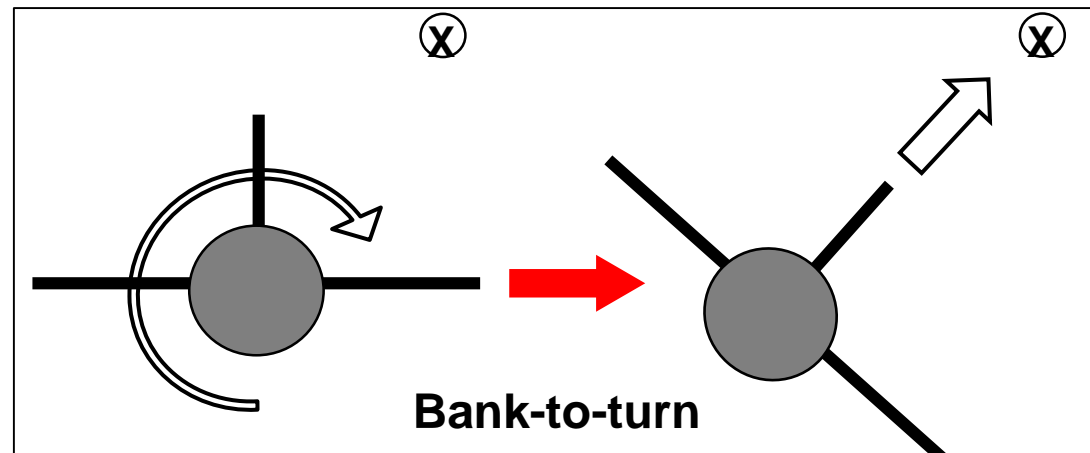
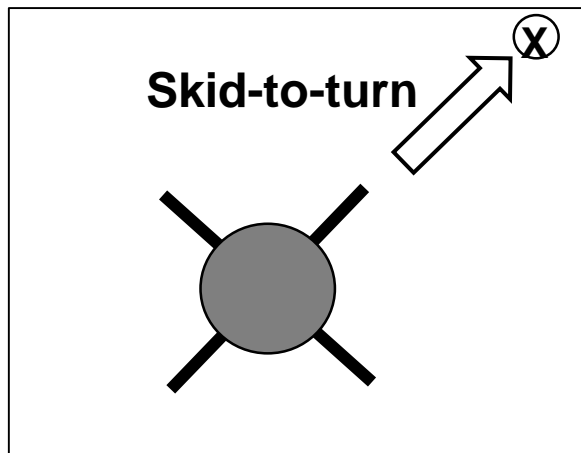
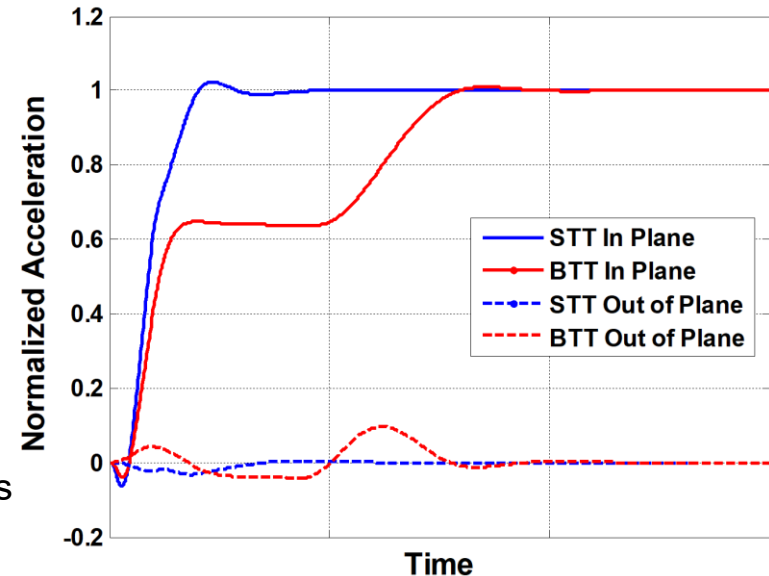
Large Flight Envelope

- Nominal vehicle response varies significantly with changes in flight condition
 - Vehicle time constant decreases as dynamic pressure increases
 - Time response characteristics such as overshoot vary based on dynamic pressure
 - As dynamic pressure is increased, a lower bound for time constant is reached based on phase constraints (from CAS bandwidth for example)
- Large variation in Mach number and Reynolds number also creates variation in vehicle response
 - This makes it very difficult (if not impossible) to find one single controller that will be robust enough to provide performance across the envelope
 - Likely requires scheduled reference models



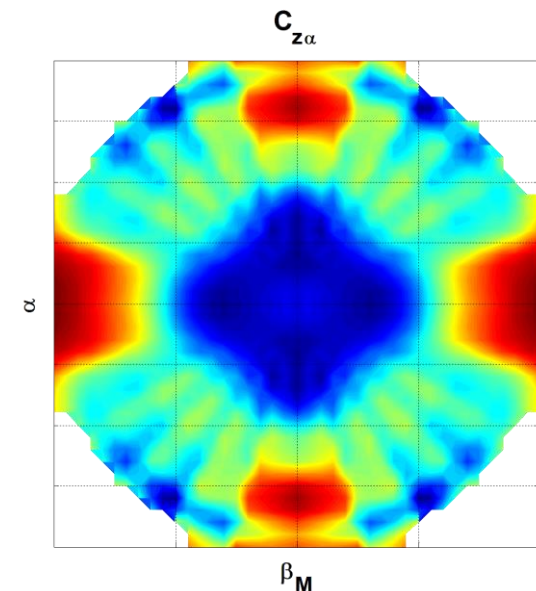
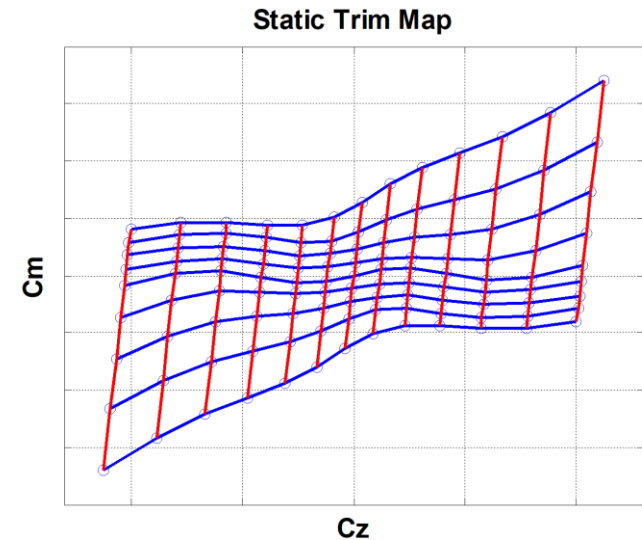
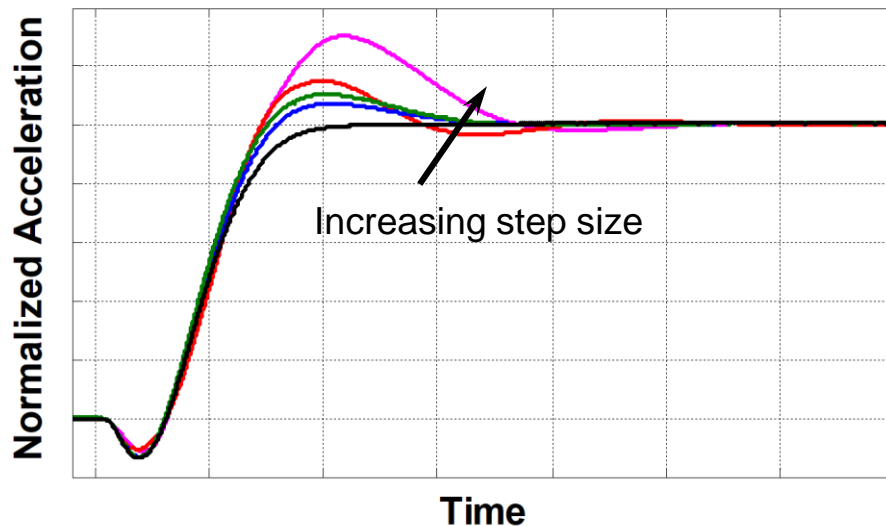
Skid-To-Turn Vs. Bank-To-Turn

- In many applications, speed of response is important to maximizing system performance
 - Typically the system may be asked to maneuver in any direction to make end-game course corrections
 - Required speed of response is dictated by uncertainties and accuracy requirements
- Cruciform systems typically perform “skid-to-turn” maneuvers
 - System maneuvers in the commanded direction
 - Faster response, but larger variation in aerodynamics based on maneuver direction
 - Prevents decoupling of pitch and lateral-directional dynamics
- Aircraft-like systems typically perform “bank-to-turn” maneuvers
 - System rolls to preferred maneuver direction
 - Results in slower speed of response, but allows for decoupling of pitch dynamics from lateral-directional dynamics



Nonlinear Aerodynamics

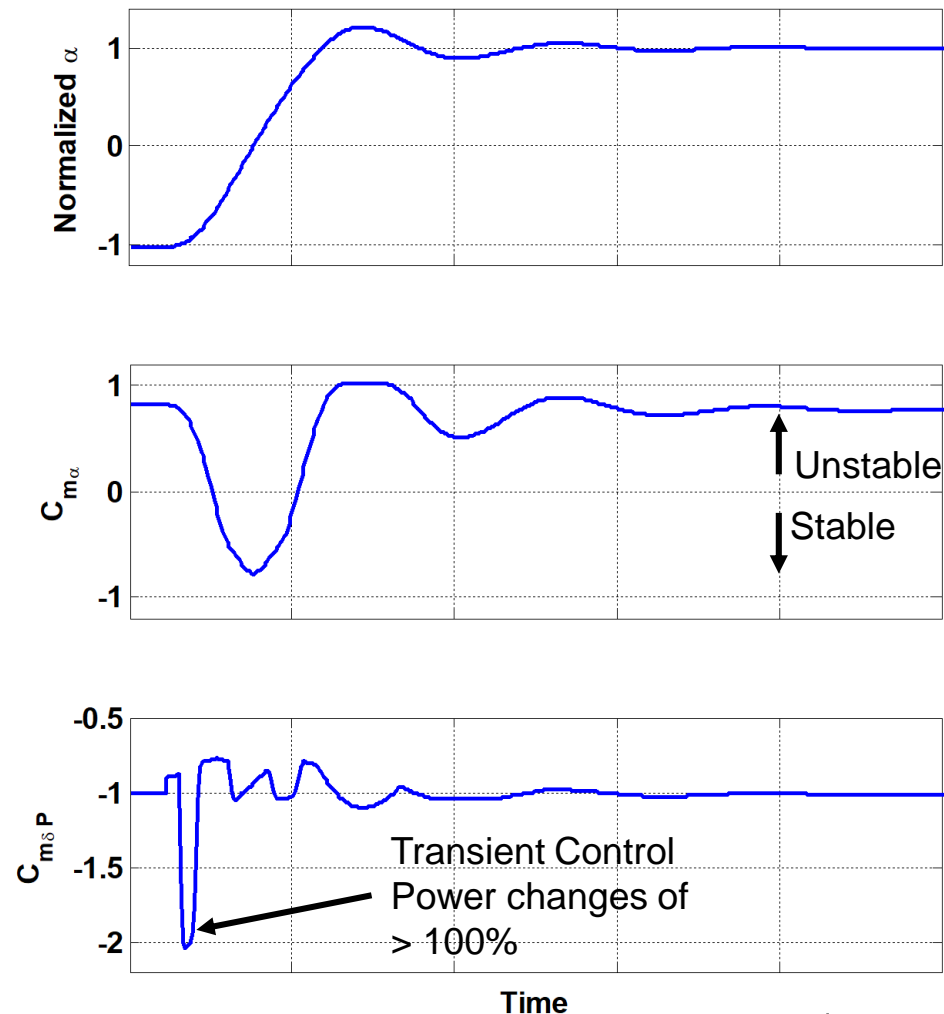
- Missile aerodynamics may experience highly nonlinear variation over the trim envelope
 - Static stability can vary over angle-of-attack and side-slip conditions
 - Control effectiveness may have nonlinear dependence on deflection
- May result in non-uniform system behavior
 - Time response characteristics dependent on maneuver size and direction
 - Transient response characteristics difficult to characterize with simplified models



Nonlinear Aerodynamics, Continued

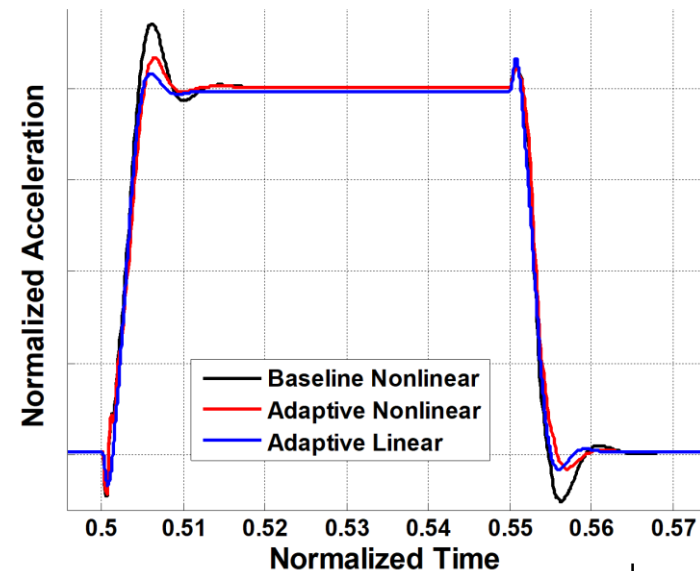
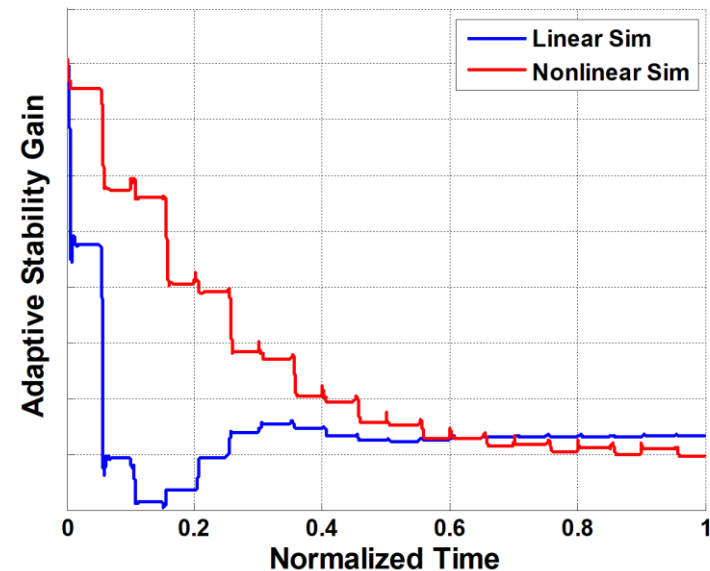
- System nonlinearities cannot be neglected for missile systems
 - High performing systems need to fly over large angle-of-attack and sideslip regimes
 - Linearized dynamics can undergo large rapid changes in missile flight regime
 - Systems may be required to quickly transition through large changes in Mach number (may transition from subsonic, through transonic, to supersonic in seconds)
- Linear modeling assumptions may be inadequate to properly capture nominal dynamics as well as uncertainties

$$M = f(\text{Mach}, \alpha, \beta, \delta R, \delta P, \delta Y)$$



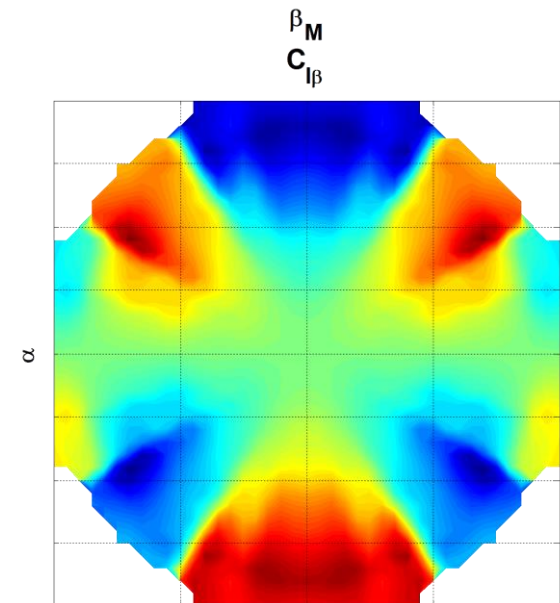
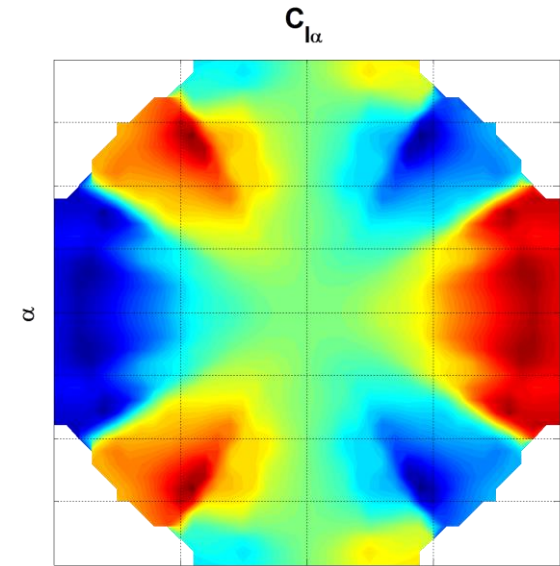
Effect on Adaptive Controller

- Adaptive laws, when designed for local linear dynamics, may not perform as well for the true nonlinear system
 - Unmodelled nonlinear control power adds dependency on deflection in local dynamics
 - Depending on regressor structure, large changes in local stability derivatives may not be captured
- Example shows convergence of state feedback adaptive law in the presence of nonlinear dynamics
 - Convergence is much slower for the nonlinear case
 - Stability converges to similar values, but response does not converge to linear model
- To handle nonlinearities, the regressor can be adjusted to something more appropriate such as a function approximator
 - Parameterization must be chosen such that $\dot{\theta} = 0$
 - Approximation error can still lead to parameter drift
- Dealing with nonlinearities in the control effectiveness is more difficult



Aerodynamic Coupling

- Typical decoupled assumption of pitch and lateral-directional equations of motion may not be applicable for missile applications
- In general, the most significant coupling occurs in the roll channel
 - Variation in angle-of-attack and side-slip can result in large variation in induced aerodynamic rolling moment
 - Roll channel inertia is typically small for missile systems when compared to pitch/yaw
 - Induced rolling moment can quickly cause large roll rates in these systems if not controlled properly
- Induced rolling moment may have large amounts of uncertainty
 - Rolling moment may have smaller signal to noise ratio in tunnel due to balance construction
 - Even slight vehicle asymmetry and CG uncertainty can result in sizable induced rolling moment

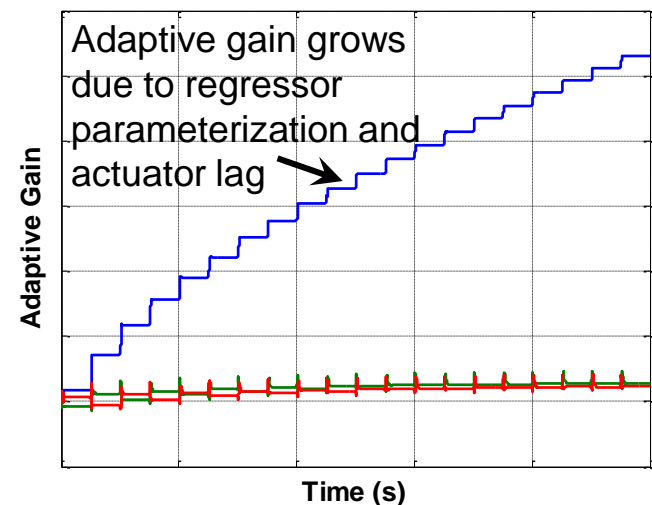
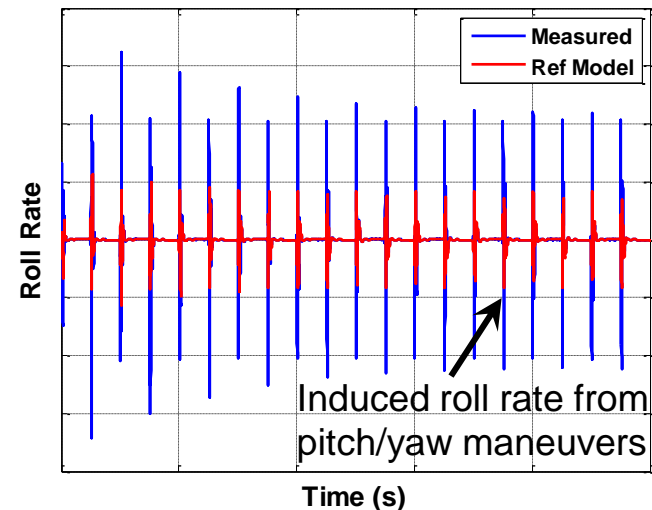


Aerodynamic Coupling, Continued

- In skid-to-turn applications, the roll channel acts as a regulator
 - Roll channel excitation enters primarily through coupling
 - Reference model will not predict behavior and will not respond (except through estimator or state predictor, depending on adaptive implementation)
- One approach may be to use a nominally decoupled reference model with linear regressor

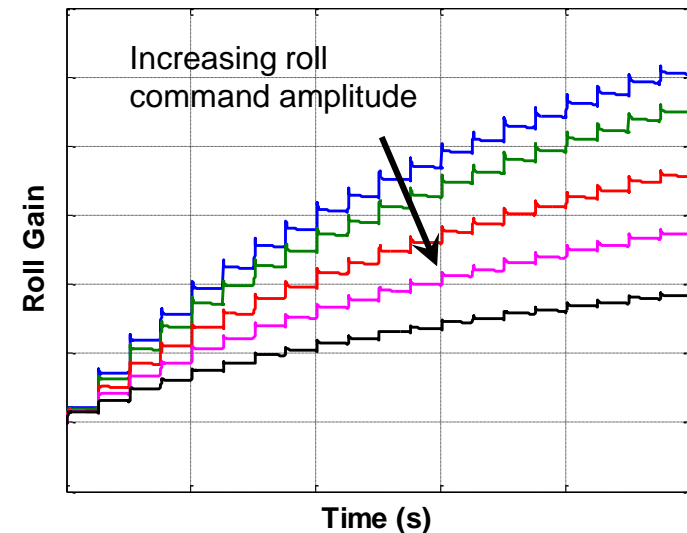
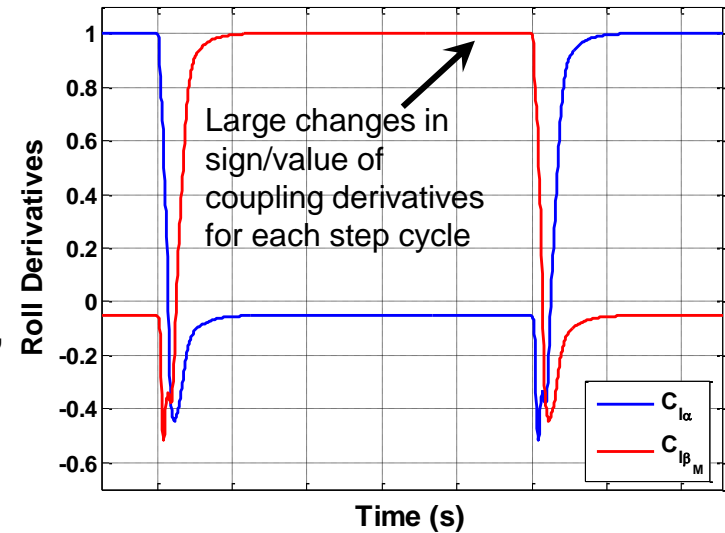
$$\Phi = [p \quad \alpha \quad \beta]^T$$

- Relies on adaptive control to remove cross-coupling
- May result in continued growth of adaptive gains (especially control power or damping type gains)



Aerodynamic Coupling, Continued

- Aerodynamic roll derivatives show large changes in behavior over the course of a single step
 - Nonlinearities in command (previous section) result in slow convergence and errors in adaptive gain
 - Reference models are typically built for command response behavior, not regulation behavior
- Choosing a better regressor can improve the response, but does not eliminate it completely
 - Large nonlinear changes in cross-coupling terms may prevent usage of a pure linear regressor
 - Feed-forward of commands into the regressor can help improve coupling – but these are largely related to angle of attack / sideslip
 - Lag due to actuators, sensors, filters, etc. prevents exact cancellation of dynamics
 - Can include nominal amount of coupling in reference model, but this is not “ideal behavior”
- Care must be taken when choosing adaptive law, learning rates, regressor, and reference model for skid-to-turn applications

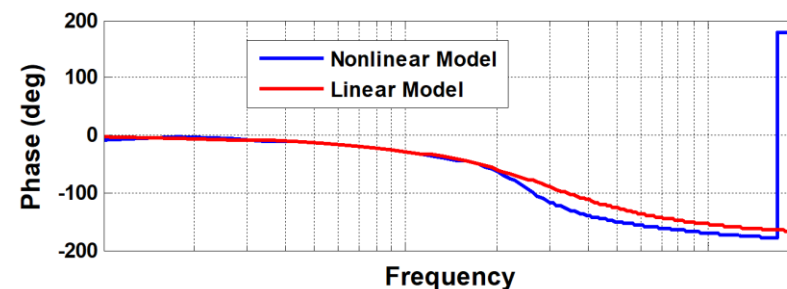
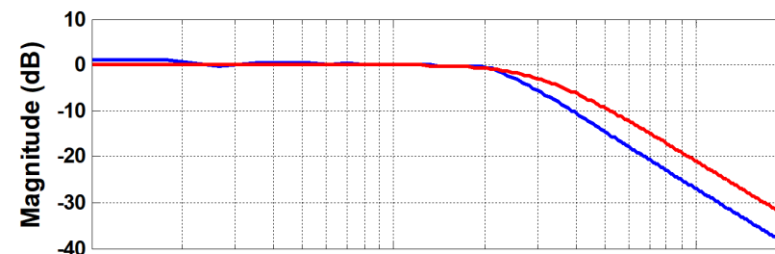
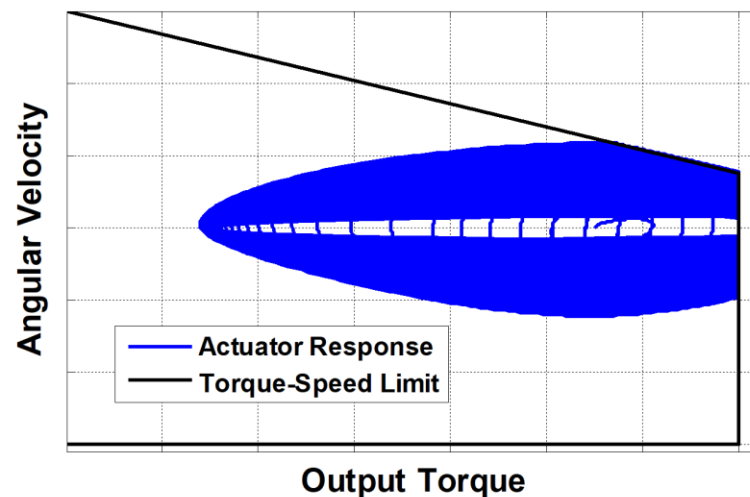
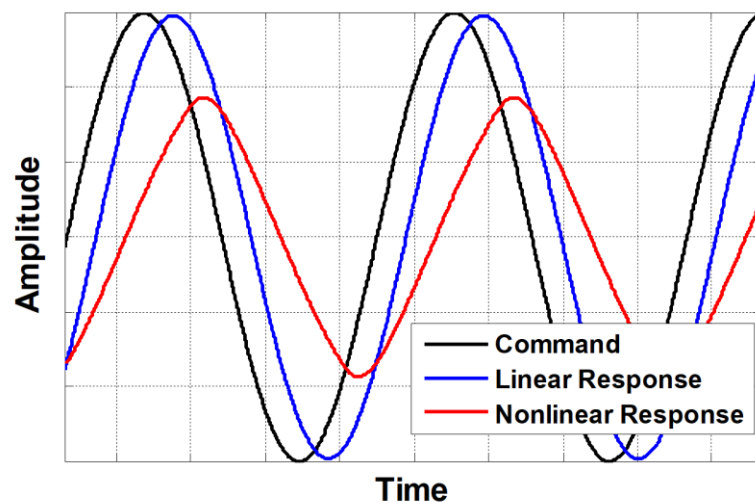


Actuator Dynamics

- The control surface actuators are typically modeled as a second order linear system

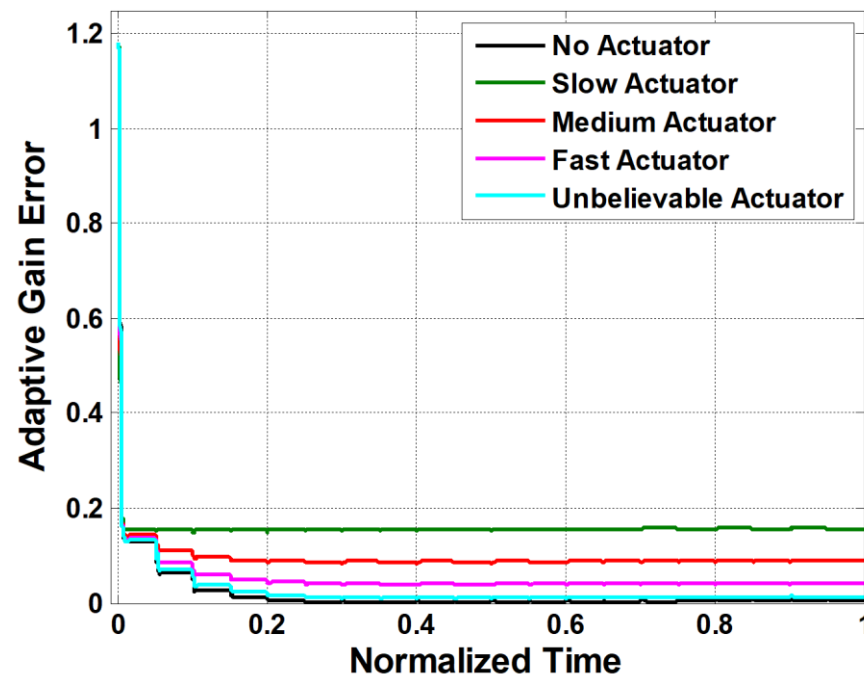
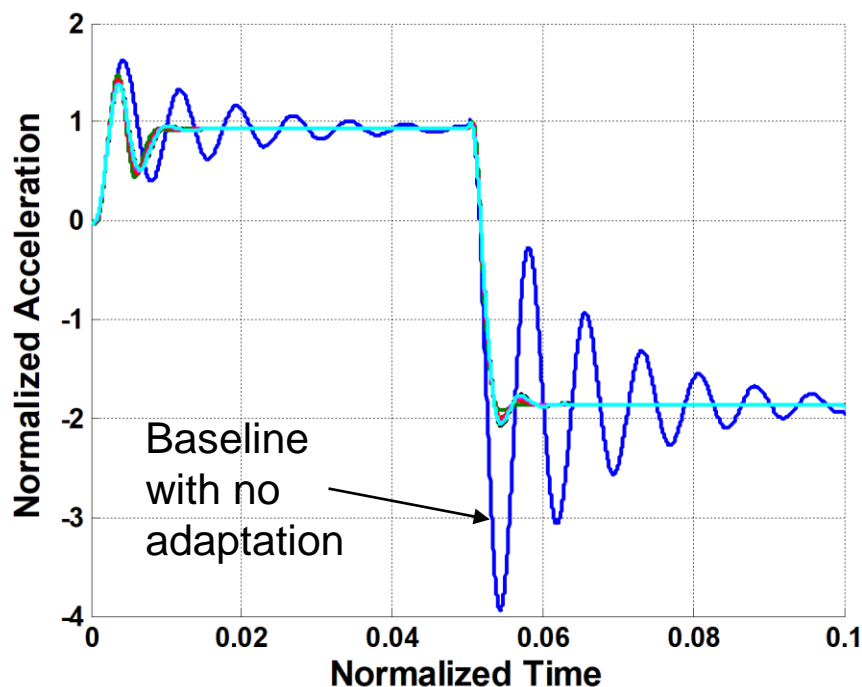
$$\delta = \frac{\omega^2}{s^2 + 2\zeta\omega s + \omega^2} \delta_c$$

- This simplified model is not always valid
 - Power capability of the actuator limits performance under load
 - Actuators have rate limitations based on load
 - Model remains valid in less stressing conditions
- Often, missile systems must operate in these regimes to maximize performance



Actuator Dynamics, Continued

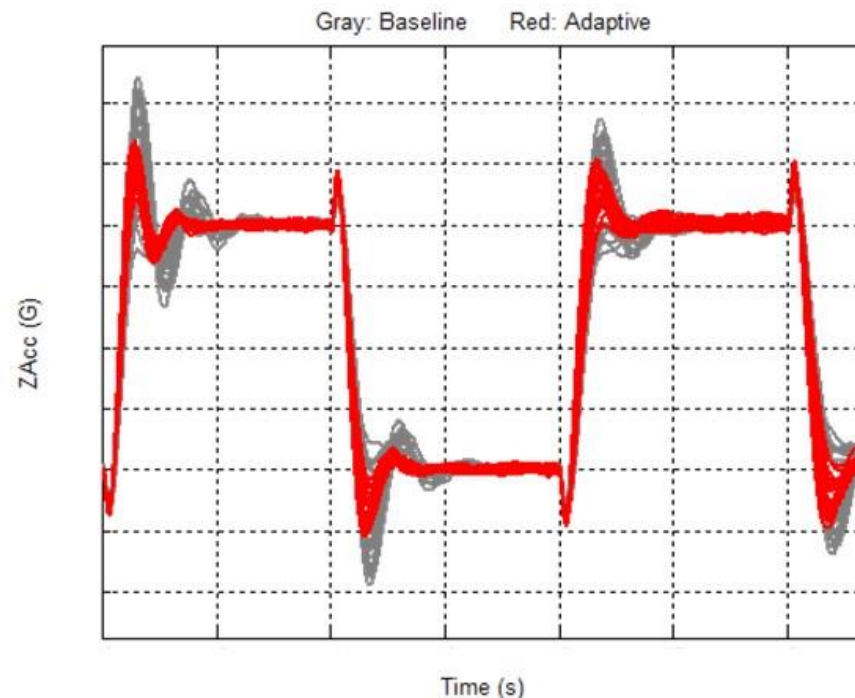
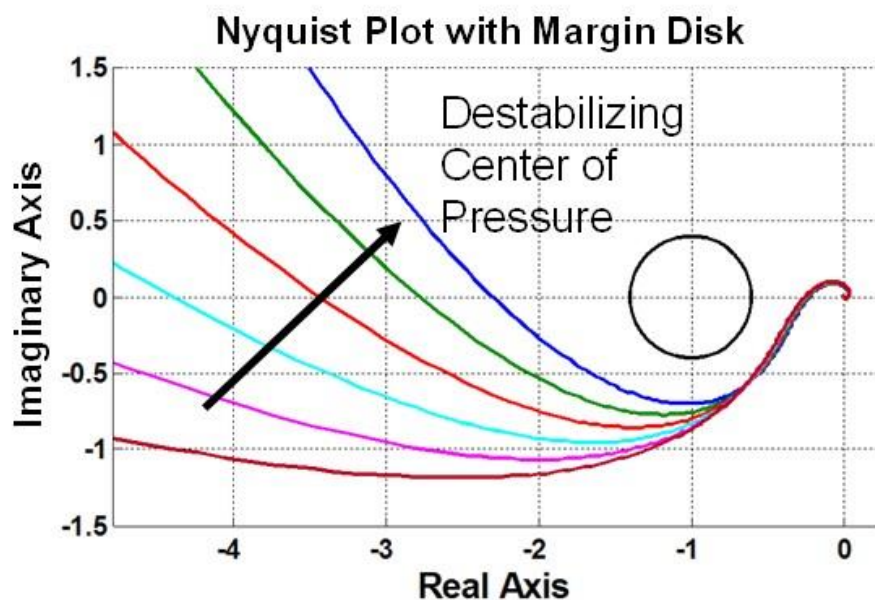
- Addition of second order actuator puts unmodelled lag between adaptive control and plant
 - Result is that uncertainty modeled as matched is now unmatched
 - Cannot directly cancel uncertainty terms due to actuator lag
 - Changes in actuator dynamics due to load, temperature, and battery state exacerbate this issue
- Actuator bandwidth affects parameter convergence
 - Slow actuator dynamics results in parameter offset even for “matched” uncertainties
 - As actuator bandwidth is increased, the system behavior tends toward that of a system with no actuator and a matched uncertainty



Benefits of Adaptive Control

Low Dynamic Pressure

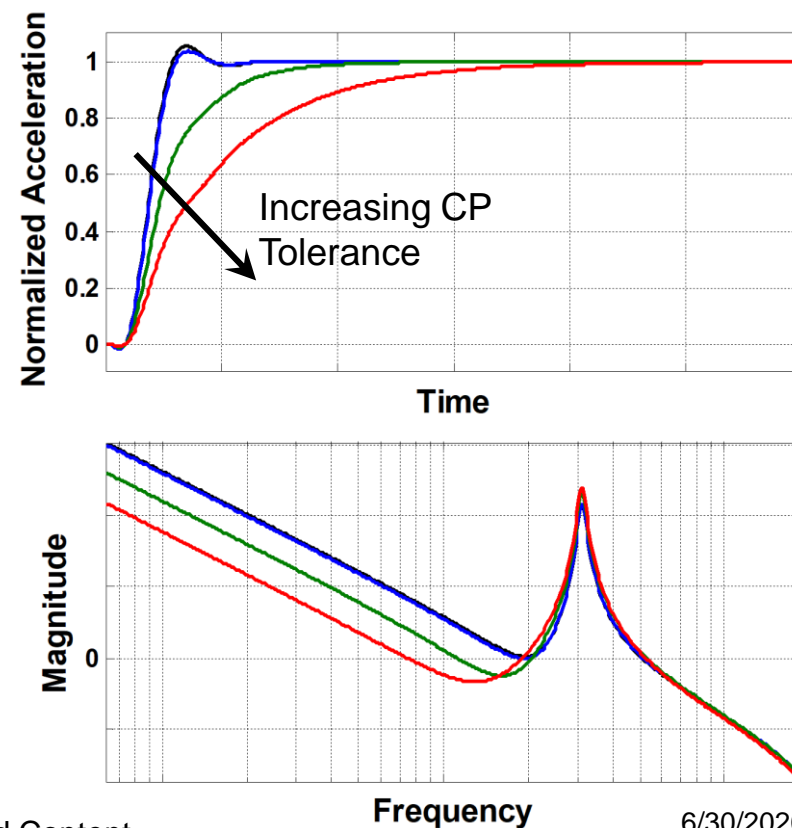
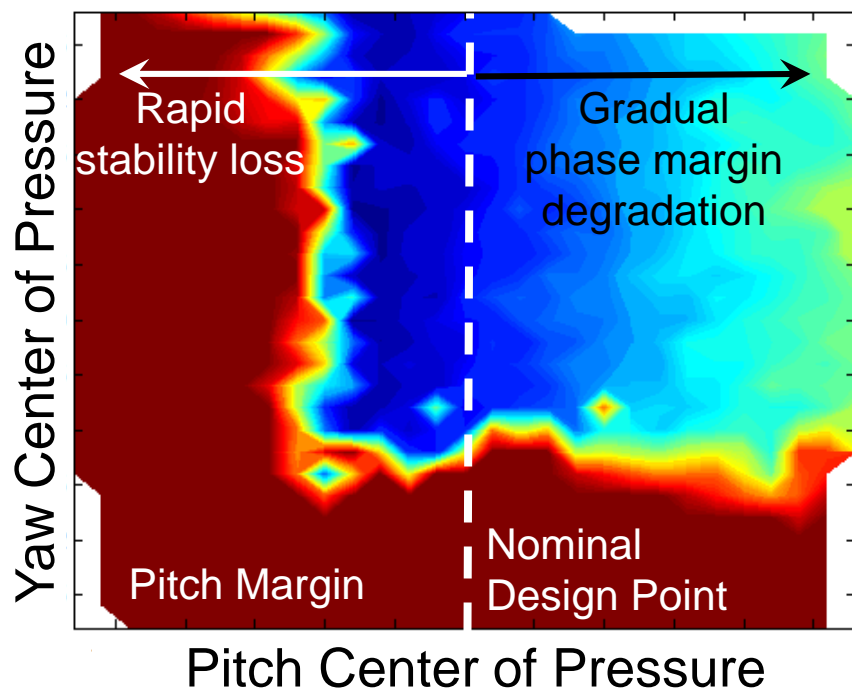
- Following plots show results for system operating at low dynamic pressure
 - System shows large amount of robustness to uncertainty in center of pressure
 - Time response shows improved uniformity with respect to uncertainty
- At lower dynamic pressures, baseline controller may have sufficient robustness to maintain linear stability
 - Actuator bandwidth requirements are typically driven by high dynamic pressure flight regime
 - System robustness may be driven by other considerations such as overall control authority
- More uniform system performance desirable for outer loop guidance



Benefits of Adaptive Control

High Dynamic Pressure

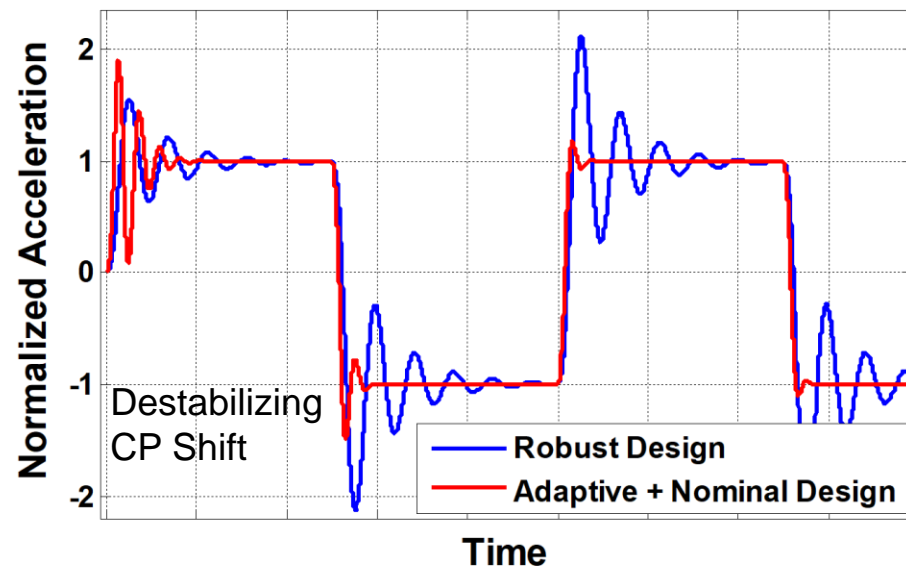
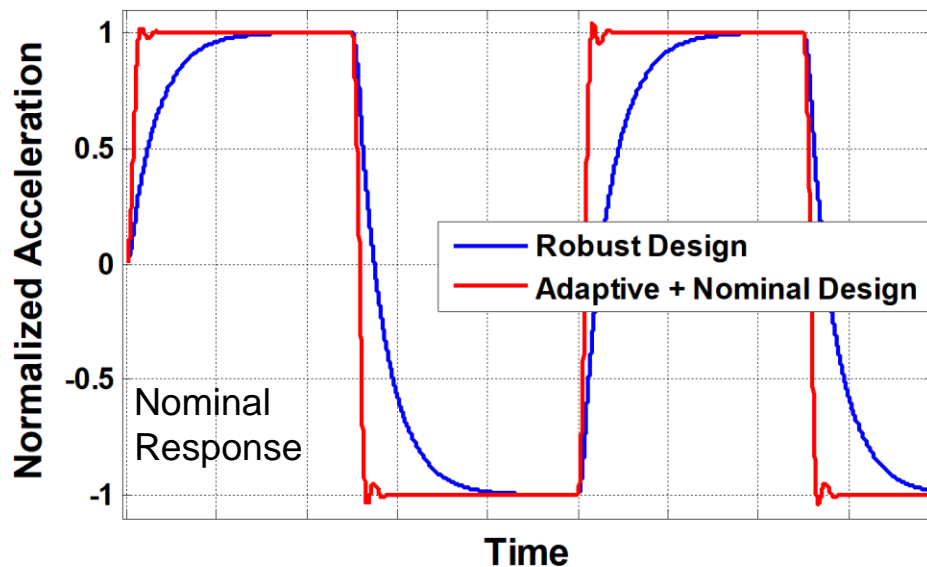
- At high dynamic pressure, design robustness limited by actuator bandwidth, structural filters, computation and communication delays, sensor dynamics, and sampling rate
- Robustness to certain types of uncertainty can be explicitly constrained in the design process, but at the cost of reduced performance



Benefits of Adaptive Control

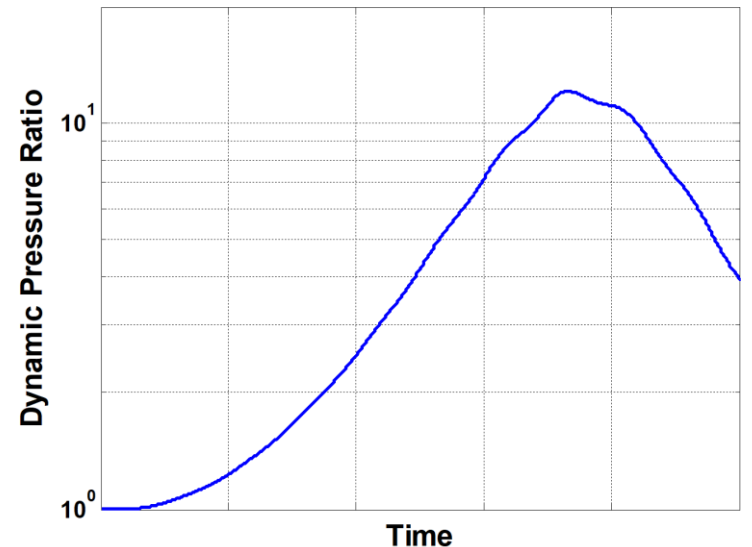
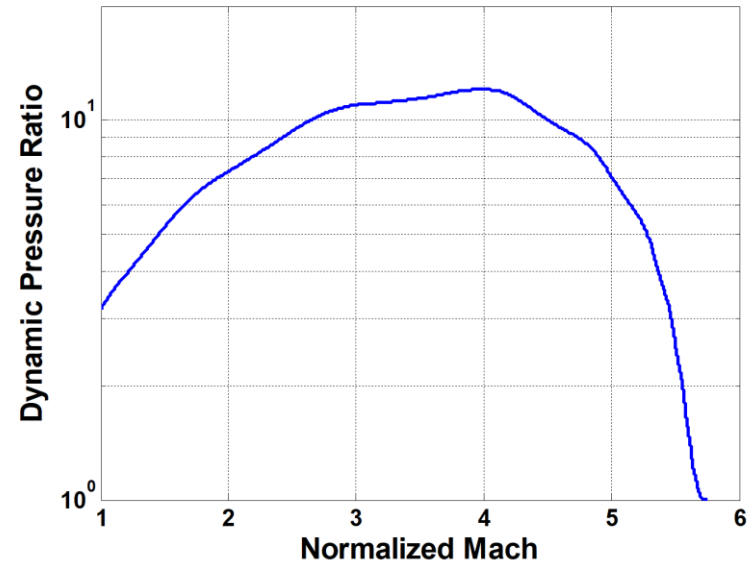
High Dynamic Pressure, Continued

- Maintaining system stability and performance with respect to aerodynamic uncertainties may not be possible with linear control
- As an example, two controllers are designed for the same system
 - First controller is designed to maintain stability to a prescribed uncertainty in center of pressure (robust design)
 - Second controller is designed for nominal performance (unstable given the prescribed center of pressure shift)
- Nominal performance of the controllers show that response time is highly degraded for the more robust design
- Augmenting the design with an adaptive control allows the nominal control design time constant to be realized with more robustness to center of pressure uncertainty



6 DOF Simulation Results

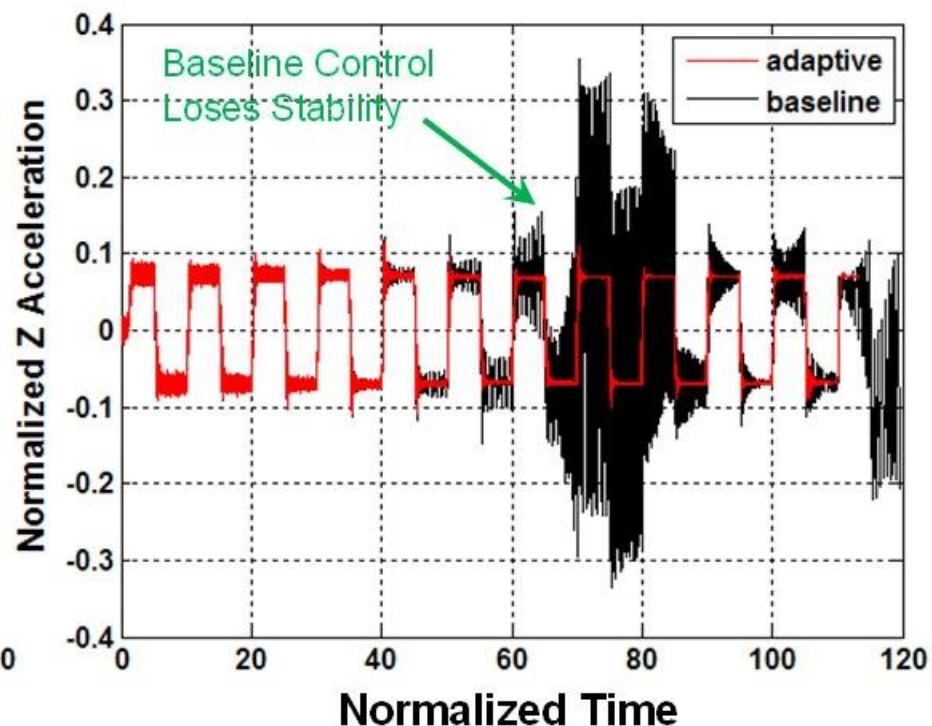
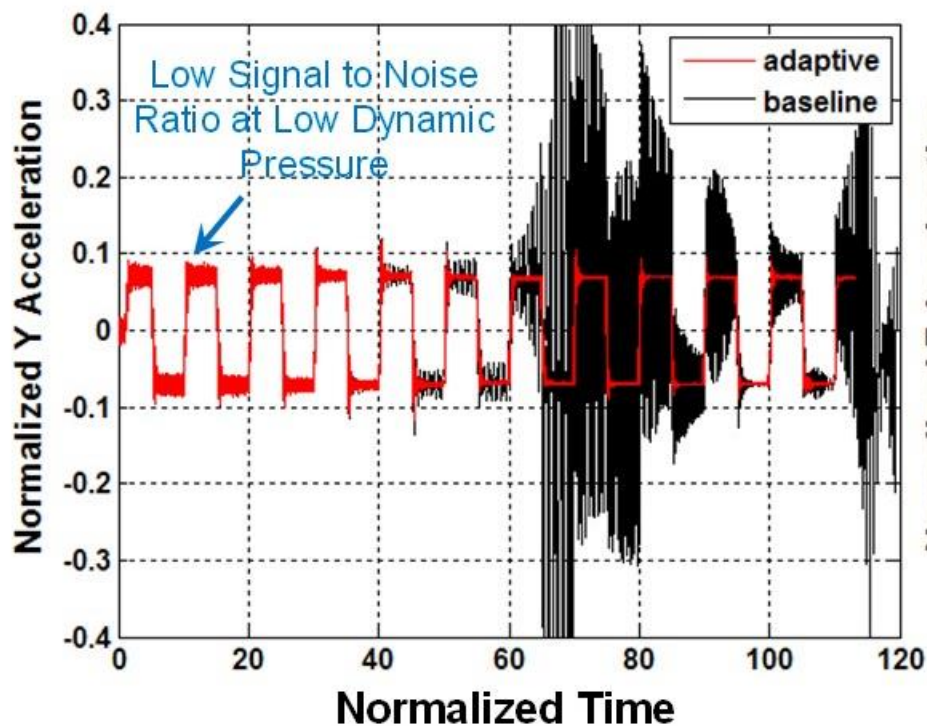
- Adaptive controller is designed for a high speed system over its entire flight regime and simulated over its trajectory
 - Fully coupled nonlinear aerodynamics model
 - High fidelity sensor models
 - Digital flight software implementation
 - High fidelity actuator model
 - Monte-Carlo uncertainty in modeling parameters
- Several scenarios considered
 - Open loop commanded flight
 - Provides frequent excitation of system
 - Size and frequency of maneuvers is controlled
 - Guided flight
 - Nearly constant commands for majority of flight
 - Main excitation occurs at beginning of flight and at terminal
- System undergoes large changes in dynamic pressure and Mach



Simulation Results

Open Loop Command Profile

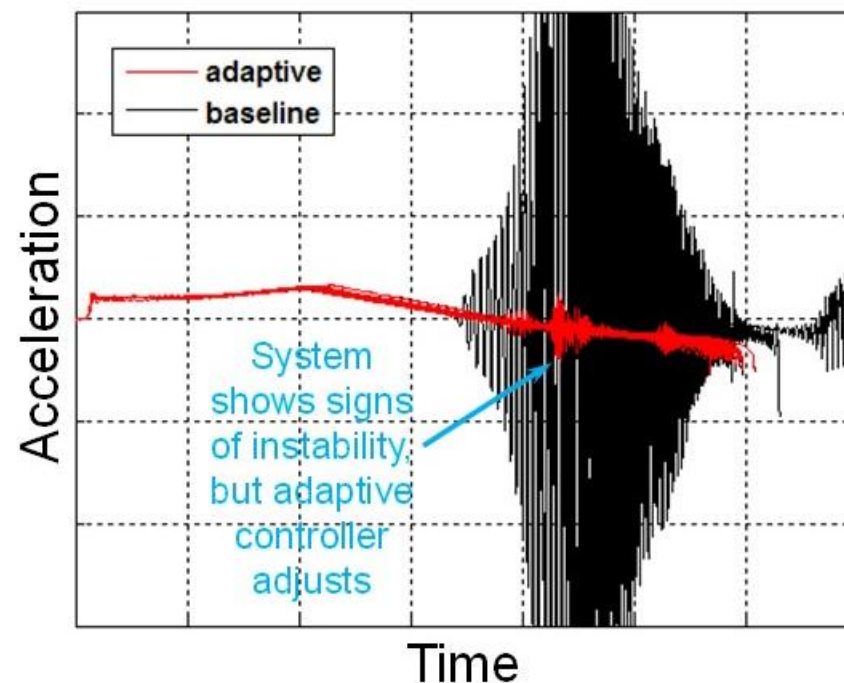
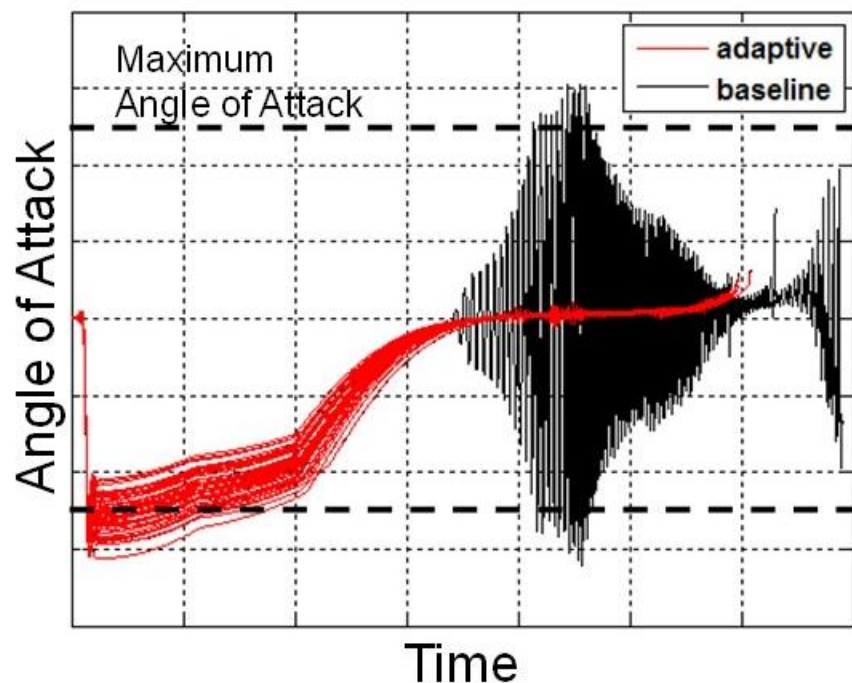
- The following simulation results are collected for a Monte-Carlo run set with the same release condition
- Acceleration doublets are commanded in pitch and yaw axes throughout the flight to demonstrate the performance of the adaptive controller
- The baseline controller loses stability for some Monte-Carlo draws at high dynamic pressure, but the adaptive controller retains stability in all cases



Simulation Results

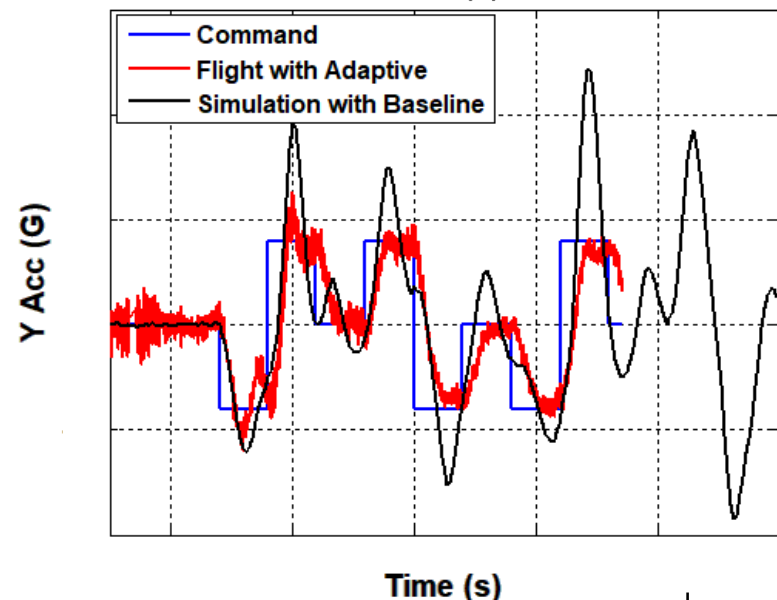
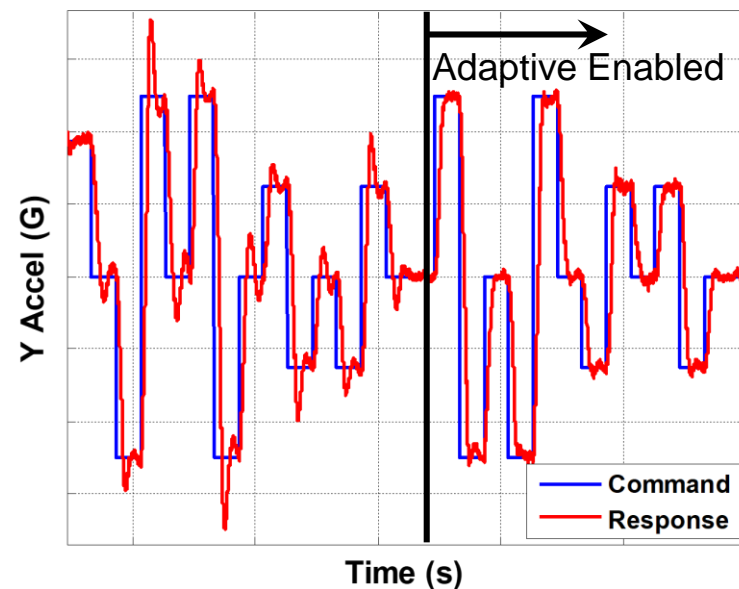
Guided Flight Profile

- The following results demonstrate the same system performance under a guided scenario
 - An initial heading correction is performed with near maximum angle of attack being commanded for the first portion of flight
 - The system then follows a slowly changing command until the final correction at the end of flight
- As with the open-loop commands, several of the runs are unstable at high dynamic pressure for the baseline controller



Flight Test Results

- Controlled test flights (pre-programmed maneuvers) as well as guided test flights have been conducted with adaptive control techniques
 - Flight control system is intentionally destabilized by performing designs for incorrect airframe properties (move CP aft for design)
 - In controlled test scenarios, the system is flown without adaptive control and then the adaptive controller is enabled
 - During guided flight test scenarios the adaptive control is flown throughout the flight
- Flight test results demonstrate accurate learning of adaptive laws
 - Guided flights require high learning rates in adaptive law since there is little excitation until final maneuver
 - Controlled flight tests show accurate parameter convergence in the presence of sufficient excitation



Summary

- Application of adaptive control methods to missile systems present significant challenges
 - Non-minimum phase
 - Fast with well-behaved transient response
 - Highly nonlinear dynamics with coupled control axes
 - Large operation envelope
- Examples have been presented that highlight the difficulties these challenges pose
- The benefits of adaptive control are demonstrated through simulation in a high fidelity environment