COOPERATIVE PLANNING AND CONTROL UNDER UNCERTAINTY

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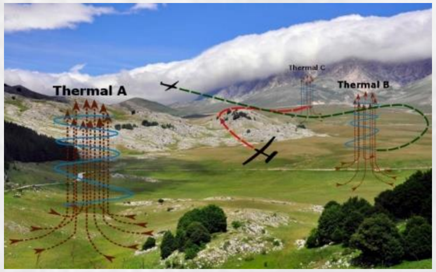
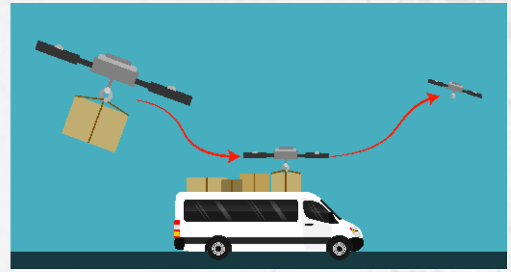
APPLICATIONS



Air sampling missions


Search and rescue missions

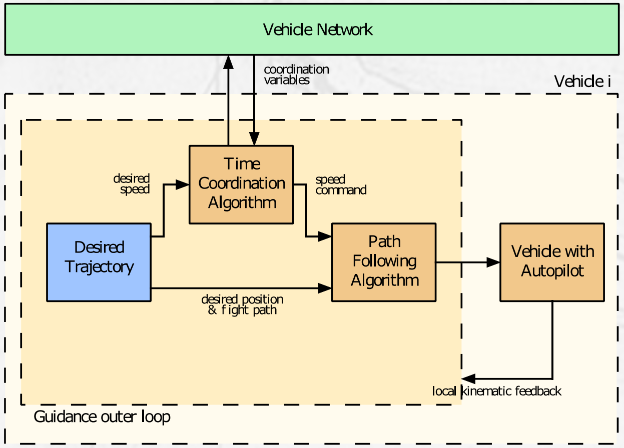

Autonomous delivery


Entertainment

Execute collision-free maneuvers and arrive at final destinations at the same time (or separated by pre-defined time intervals)
React in real time to changes

- Applications for multiple vehicles:
  - Cooperative collision avoidance
  - Cooperative target tracking

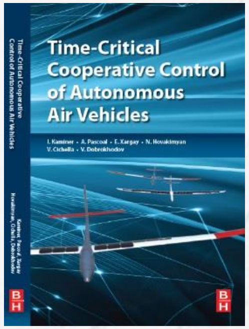
DECOUPLING SPACE AND TIME



❖ Cooperative control architecture

- Motion Planning (open-loop): generate geometric paths
  - Nonlinear optimal control problem, direct methods
- Coordination (closed-loop): adjust speeds to meet temporal requirements
  - Consensus, graph theory, nonlinear systems

- Advantages
  - Flexibility
    - general problem formulation – motion planning
  - Robustness
    - Reactively adjust the speed online for safety/mission objectives
    - Low exchange of information
    - Mild assumptions on coordination network



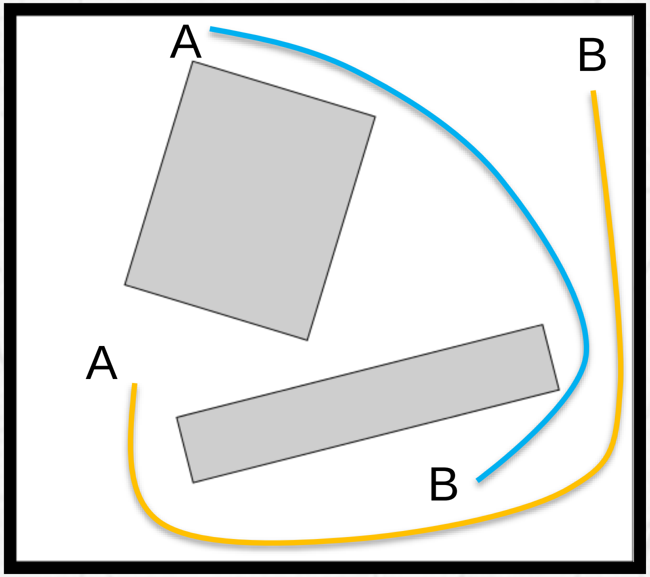
Kaminer, I., Pascoal, A. M., Xargay, E., Hovakimyan, N., Cichella, V., & Dobrokhodov, V. (2017). *Time-Critical cooperative control of autonomous air vehicles*. Butterworth-Heinemann.

Optimal motion planning

OCP: determine $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$

that minimize $$E(\boldsymbol{x}(0), \boldsymbol{x}(t_f)) + \int_{\Theta} \left(\int_T F(\boldsymbol{x}(t), \boldsymbol{u}(t), \theta) dt \right) \boldsymbol{p}(\theta) d\theta$$

subject to
$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \quad \forall t \in [0, t_f]$$
$$\boldsymbol{e}(\boldsymbol{x}(0), \boldsymbol{x}(t_f)) = \boldsymbol{0}$$
$$\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \leq \boldsymbol{0}, \quad \forall t \in [0, t_f]$$



Optimal motion planning

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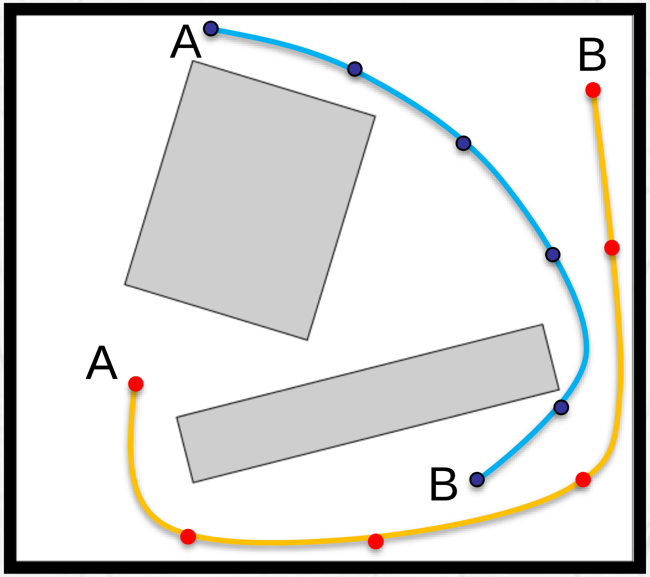
$$\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \le \boldsymbol{0}, \quad \forall t \in [0, t_f]$$

$$\bar{\boldsymbol{t}} = [t_0, \ldots, t_N] \qquad \bar{\boldsymbol{x}} = [\boldsymbol{x}_0, \ldots, \boldsymbol{x}_N] \qquad \bar{\boldsymbol{u}} = [\boldsymbol{u}_0, \ldots, \boldsymbol{u}_N]$$

NLP: determine $\bar{\boldsymbol{x}}$ and $\bar{\boldsymbol{u}}$

that minimize $$E(\boldsymbol{x}_0, \boldsymbol{x}_N) + \sum_{i=0}^{M} w_i \sum_{j=0}^{N} w_j F(\boldsymbol{x}_j, \boldsymbol{u}_j, \theta_i) \boldsymbol{p}(\theta_i)$$

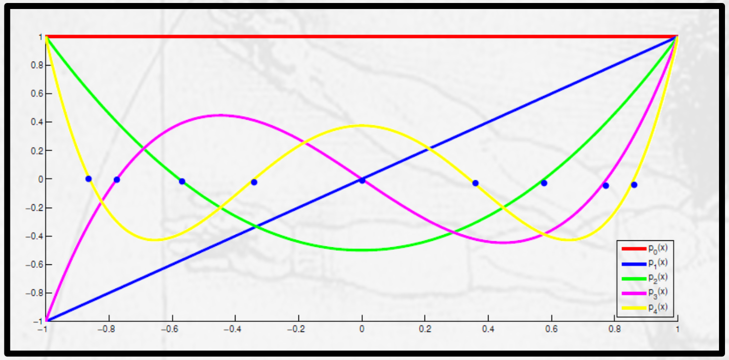
subject to
$$\left\| \sum_{j=0}^{N} D_{ij} \boldsymbol{x}_j - \boldsymbol{f}(\boldsymbol{x}_i, \boldsymbol{u}_i) \right\| \le N^{-\delta}$$
$$\|\boldsymbol{e}(\boldsymbol{x}_0, \boldsymbol{x}_N)\| \le N^{-\delta}$$
$$\boldsymbol{h}(\boldsymbol{x}_i, \boldsymbol{u}_i) \le \boldsymbol{1} N^{-\delta}$$



Approximate - Solve - Interpolate

LGL Pseudospectral

- Legendre-Gauss-Lobatto (LGL) nodes:



$$t_0 = -1\,,\; t_N = 1\,,\ \text{and } t_i \text{ are roots of } q(t) = \dot{L}_N$$

$$L_N(x) = \frac{1}{2^N N!}\frac{d^N}{dx^N}[(x^2-1)^N]$$

- Lagrange interpolation:

$$\boldsymbol{x}(t) \approx \boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{x}(t_k)\ell_k(t) \qquad \ell_i = \prod_{k=0,k\neq i}^{N} \frac{t-t_k}{t_i-t_k}$$

- Differentiation:

$$\dot{\boldsymbol{x}}(t_k) \approx \dot{\boldsymbol{x}}_N(t_k) = \sum_{i=0}^{N} \boldsymbol{x}(t_i)\boldsymbol{D}_{ki} \qquad \boldsymbol{D} = \begin{bmatrix} \dot{\ell}_0(t_0) & \dot{\ell}_1(t_0) & \cdots & \dot{\ell}_N(t_0) \\ \dot{\ell}_0(t_1) & \dot{\ell}_1(t_1) & \cdots & \dot{\ell}_N(t_1) \\ \vdots & \vdots & \ddots & \vdots \\ \dot{\ell}_0(t_N) & \dot{\ell}_1(t_N) & \cdots & \dot{\ell}_N(t_N) \end{bmatrix}$$

- Gaussian quadrature:

$$\int_{-1}^{1} \boldsymbol{x}(t)dt \approx \sum_{i=0}^{N} w_i \boldsymbol{x}(t_i) \qquad w_0 = w_N = \frac{2}{N(N+1)}\,, \quad w_i = \frac{2}{N(N+1)[L_N(t_i)]^2}$$

Ross, I. Michael, and Fariba Fahroo. "Legendre pseudospectral approximations of optimal control problems." In *New trends in nonlinear dynamics and control and their applications*, pp. 327-342. Springer, Berlin, Heidelberg, 2003.

LGL Pseudospectral

❑ Advantages

- Lagrange interpolation at Legendre nodes is robust – *for sufficiently smooth solutions*
- Consistency analysis [Polak, 1997]
  - NLP is feasible
  - Solutions to NLP converge to solutions to OCP
  - The proof relies on orthogonal collocation property of Lagrange interpolants

$$\boldsymbol{x}(t) \approx \boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{x}(t_k)\ell_k(t) \qquad \ell_i(t_j) = \begin{cases} 1 & \text{if } i = j \\ 0 & \text{if } i \neq j \end{cases} \implies \boldsymbol{x}_N(t_i) = \boldsymbol{x}(t_i)$$

- High rate of convergence

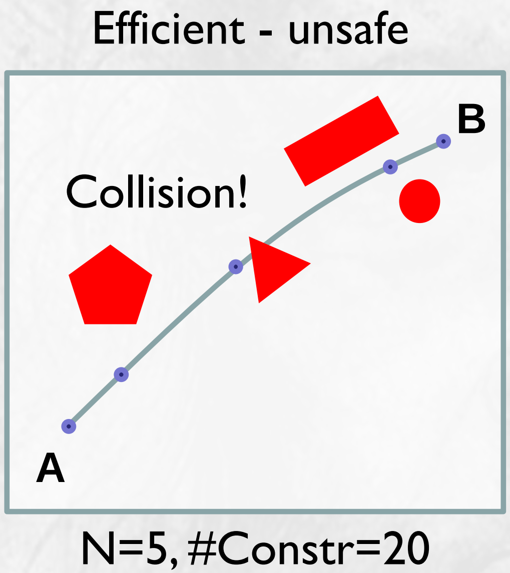
$$\boldsymbol{x} \in W^{m,\infty} \implies ||\boldsymbol{x}_N(t) - \boldsymbol{x}(t)||_{L^2} \leq \frac{C}{N^m}$$

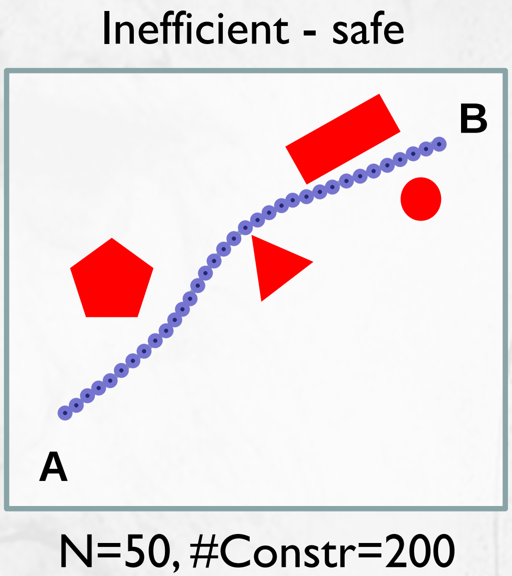
❑ Main disadvantage

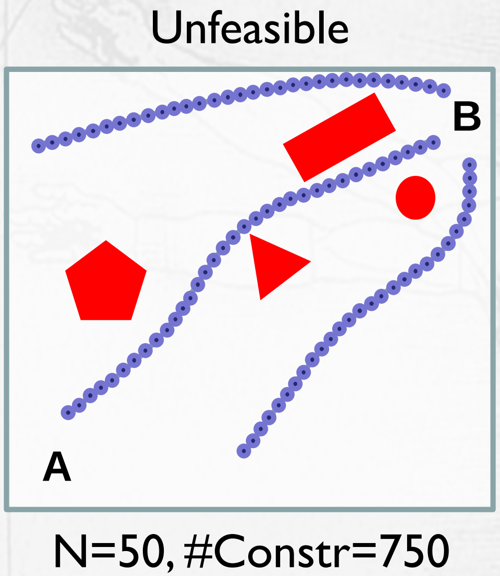
- Constraints can be imposed only at the nodes
  - Efficient VS Constraints Satisfaction (SAFETY)

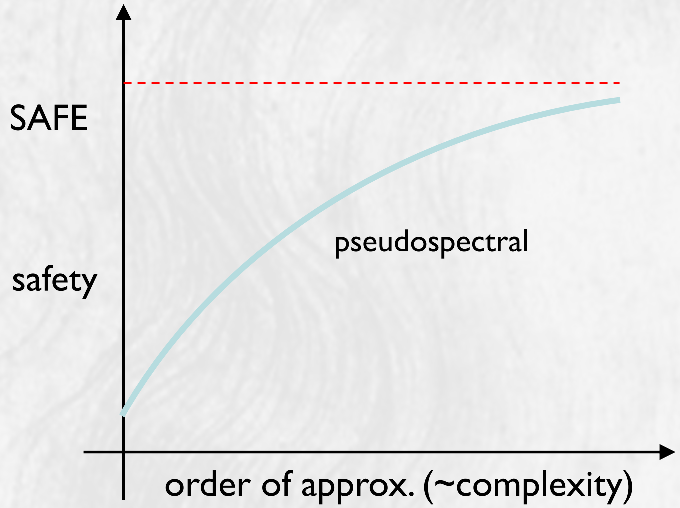
E. Polak, “Optimization: Algorithms and consistent approximations,” 1997, Springer Verlage Publications.

Efficiency VS Safety







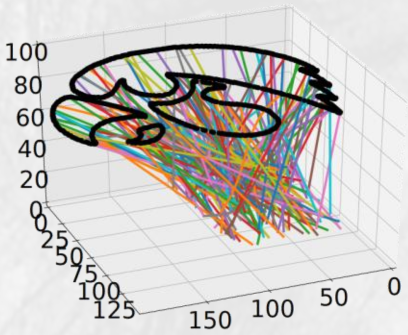


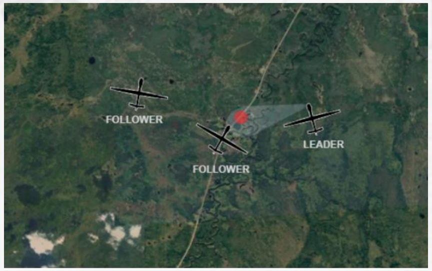
Efficiency VS Safety

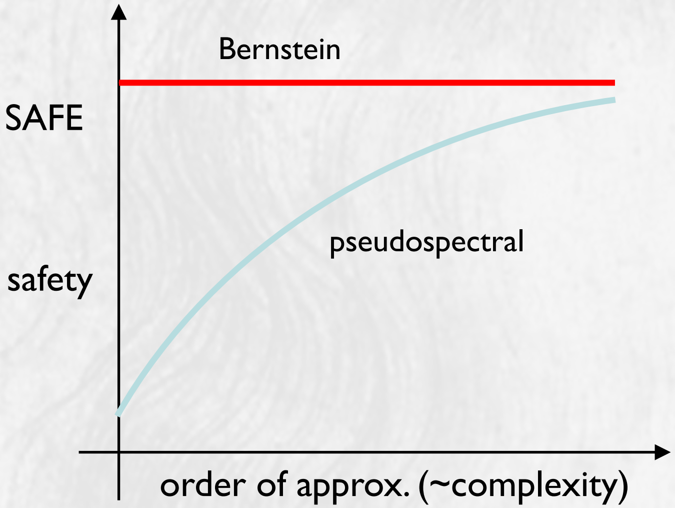
Scalability
(swarm)

Scalability
(air traffic)

Real-time
OC







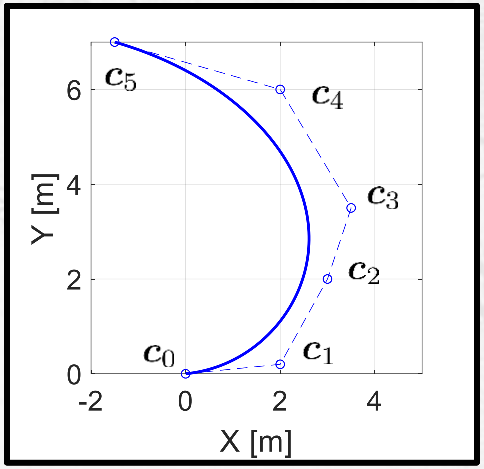
We seek a class of polynomials with geometric properties that can be exploited in satisfying the set of imposed constraints:

Bernstein polynomials

Bernstein polynomials

A degree *n* Bernstein polynomial is given by

$$\boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{c}_k \, b_{k,N}(t)$$



where

- $b_{k,N}(t)$ are the Bernstein polynomial basis

$$b_{k,N} = \binom{N}{k} t^N (t_f - t)^{N-k}, \quad t \in [0, t_f]$$

- $\mathbf{c}_k \in \mathbb{R}^3$ are the *Bernstein coefficients*

Sergei Bernstein (1880-1968)

Paul de Casteljau (1930)

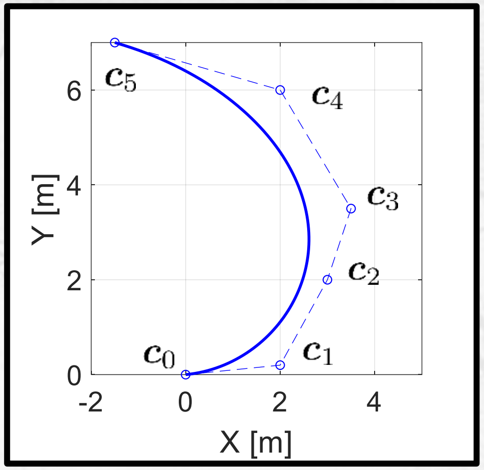
Pierre Bézier (1910-1999)

Farouki, Rida T. "The Bernstein polynomial basis: A centennial retrospective." *Computer Aided Geometric Design* 29, no. 6 (2012): 379-419.

Bernstein polynomial approximation

A degree *N* Bernstein polynomial is given by

$$\boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{c}_k \, b_{k,N}(t)$$



- Bernstein approximation

$$t_j = 0, \quad j = 0, \ldots, N, \qquad \boldsymbol{c}_j = \boldsymbol{x}(t_j)$$

$$\boldsymbol{x}(t) \approx \boldsymbol{x}_N(t) = \sum_{j=0}^{N} \boldsymbol{c}_j b_{j,N}(t)$$

- Differentiation

$$\dot{\boldsymbol{x}}(t) \approx \dot{\boldsymbol{x}}_N(t) = \sum_{j=0}^{N-1} \left(\sum_{i=0}^{N} \boldsymbol{D}_{ji} \boldsymbol{c}_i \right) b_{j,N}(t) \qquad \boldsymbol{D} = \begin{bmatrix} -\frac{N}{t_f} & 0 & \cdots & 0 \\ \frac{N}{t_f} & \ddots & \cdots & \vdots \\ 0 & \ddots & \ddots & 0 \\ \vdots & \ddots & \ddots & -\frac{N}{t_f} \\ 0 & \cdots & \cdots & \frac{N}{t_f} \end{bmatrix}$$

- Quadrature

$$\int_0^{t_f} \boldsymbol{x}(t)dt \approx \sum_{i=0}^{N} w_i \boldsymbol{x}(t_i) \qquad w_i = \frac{t_f}{N+1}$$

Optimal motion planning

OCP: determine $\boldsymbol{x}(t)$ and $\boldsymbol{u}(t)$

that minimize $$E(\boldsymbol{x}(0), \boldsymbol{x}(t_f)) + \int_\Theta \left(\int_T F(\boldsymbol{x}(t), \boldsymbol{u}(t), \theta) dt \right) \boldsymbol{p}(\theta) d\theta$$

subject to
$$\dot{\boldsymbol{x}} = \boldsymbol{f}(\boldsymbol{x}(t), \boldsymbol{u}(t)), \quad \forall t \in [0, t_f]$$
$$\boldsymbol{e}(\boldsymbol{x}(0), \boldsymbol{x}(t_f)) = \boldsymbol{0}$$
$$\boldsymbol{h}(\boldsymbol{x}(t), \boldsymbol{u}(t)) \leq \boldsymbol{0}, \quad \forall t \in [0, t_f]$$

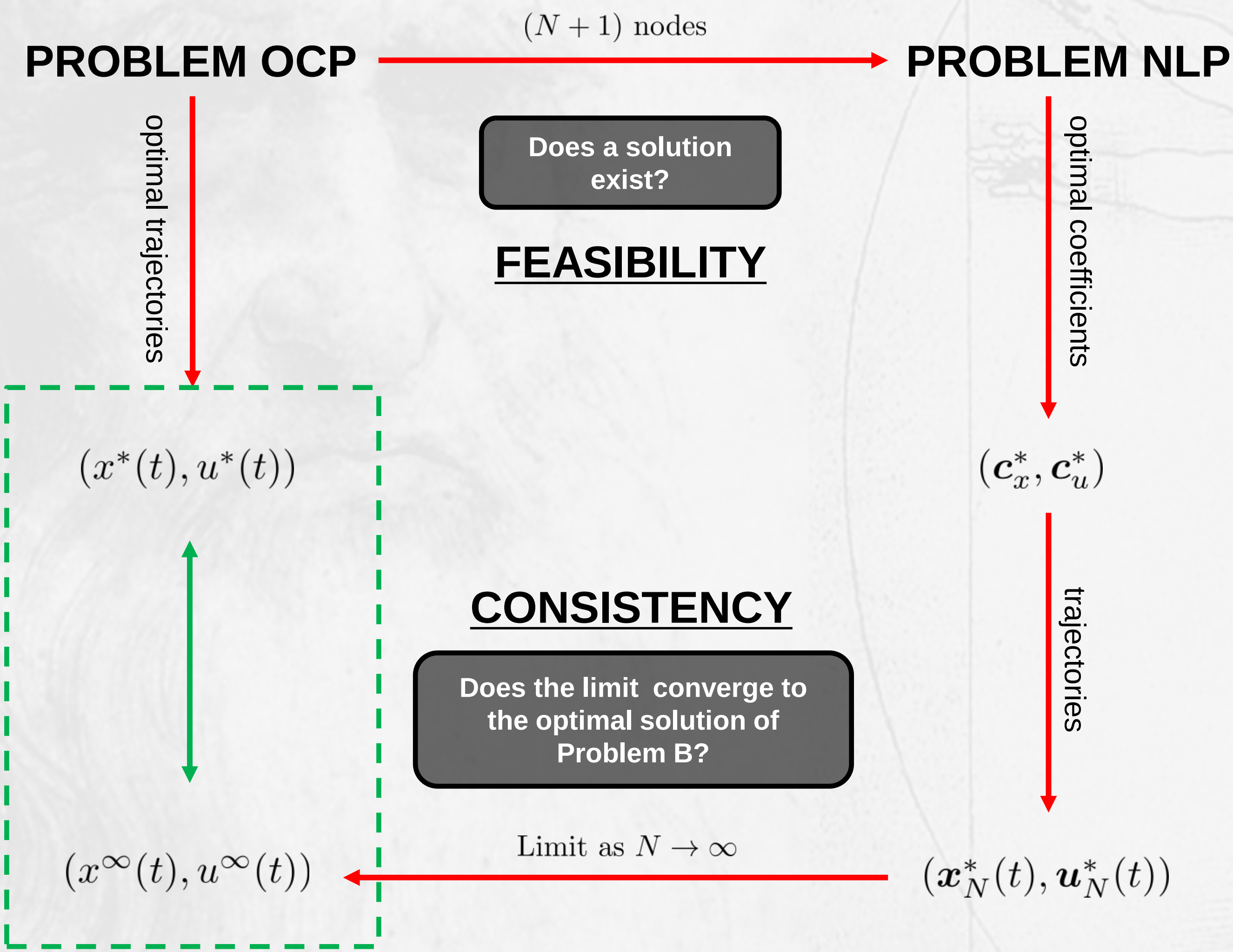
$$\boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{x}_k \, b_{k,N}(t) \qquad \boldsymbol{u}_N(t) = \sum_{k=0}^{N} \boldsymbol{u}_k \, b_{k,N}(t)$$

NLP: Let $0 < \delta_P < 1$. Determine $\boldsymbol{x}_k$ and $\boldsymbol{u}_k$, $k = 0, \ldots, N$

that minimize $$E(\boldsymbol{x}_0, \boldsymbol{x}_N) + \sum_{i=0}^{M} w_i \sum_{j=0}^{N} w_j F(\boldsymbol{x}_j, \boldsymbol{u}_j, \theta_i) \boldsymbol{p}(\theta_i)$$

subject to
$$\left\| \sum_{i=0}^{N} D_{ji} \boldsymbol{x}_i - \boldsymbol{f}(\boldsymbol{x}_j, \boldsymbol{u}_j) \right\| \leq N^{-\delta_P} \quad \forall j = 0, \ldots, N$$
$$\boldsymbol{e}(\boldsymbol{x}_0, \boldsymbol{x}_N) = \boldsymbol{0}$$
$$\boldsymbol{h}(\boldsymbol{x}_j, \boldsymbol{u}_j) \leq N^{-\delta_P} \boldsymbol{1}, \quad \forall j = 0, \ldots, N$$

Numerical Properties - Consistency



Cichella, V., Kaminer, K., Walton, C., Hovakimyan, N., Pascoal, A., “Optimal Multi-Vehicle Motion Planning using Bernstein Approximants,” 2020, IEEE Transactions on Automatic Control, Accepted

Convergence Rate

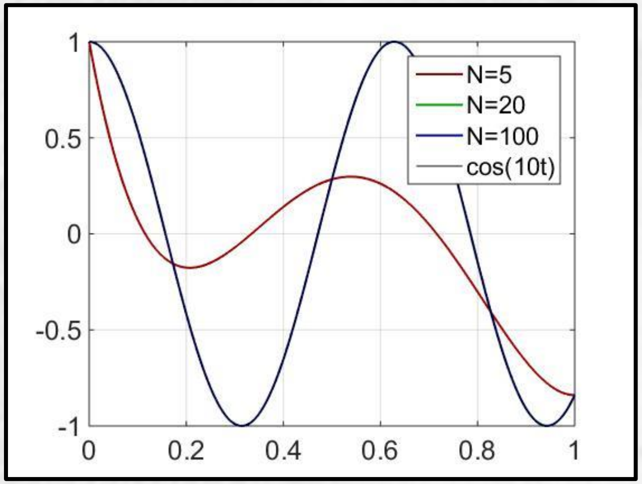
"The fact seems to have precluded any numerical application of Bernstein polynomials from having been made. Perhaps they will find application when the properties of the approximant in the large are of more importance than the closeness of the approximation."

For $m \geq 2$

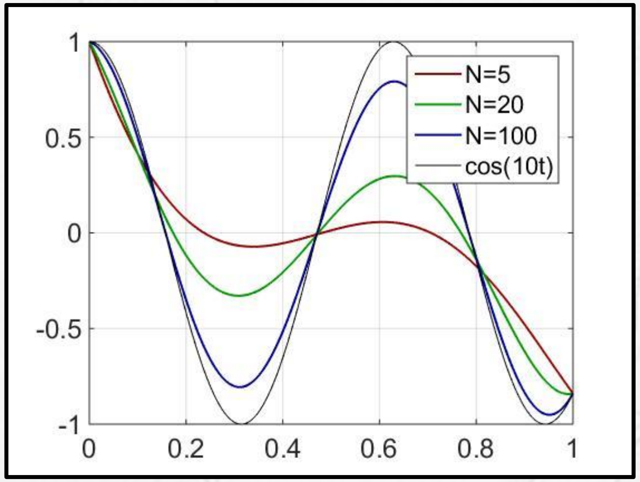
$$\boldsymbol{x} \in W^{m,\infty} \implies ||\boldsymbol{x}_N(t) - \boldsymbol{x}(t)||_{L^2} \leq \frac{C}{N^m}$$

$$\boldsymbol{x} \in \mathcal{C}^m \implies ||\boldsymbol{x}_N(t) - \boldsymbol{x}(t)|| \leq \frac{C}{N}$$

Lagrange interpolation (Legendre nodes)



Bernstein Approximation (equidistant nodes)



Davis, Philip J. *Interpolation and approximation*. Courier Corporation, 1975.

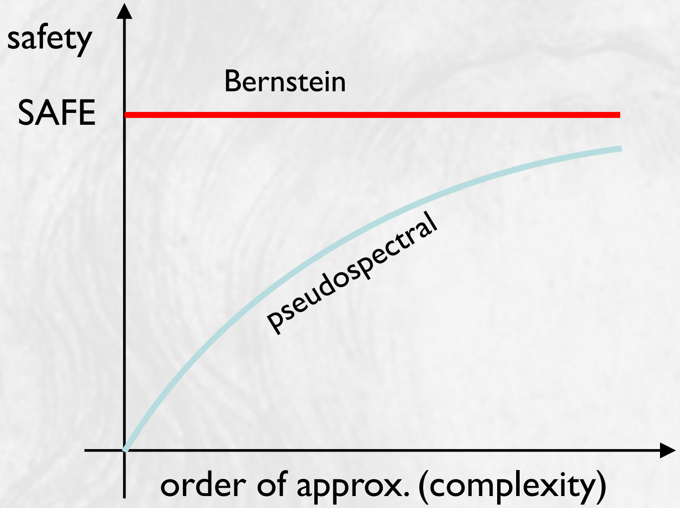
Convergence Rate

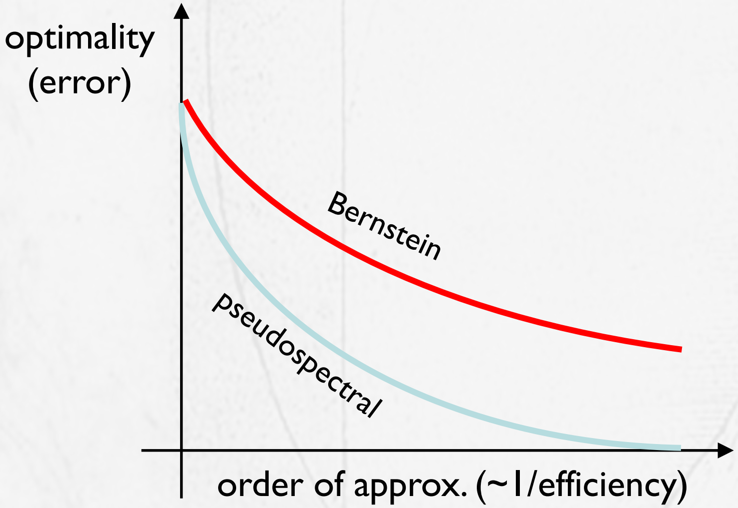
"The fact seems to have precluded any numerical application of Bernstein polynomials from having been made. Perhaps they will find application when the properties of the approximant in the large are of more importance than the closeness of the approximation."

For $m \geq 2$

$$\boldsymbol{x} \in W^{m,\infty} \implies ||\boldsymbol{x}_N(t) - \boldsymbol{x}(t)||_{L^2} \leq \frac{C}{N^m}$$

$$\boldsymbol{x} \in C^m \implies ||\boldsymbol{x}_N(t) - \boldsymbol{x}(t)|| \leq \frac{C}{N}$$

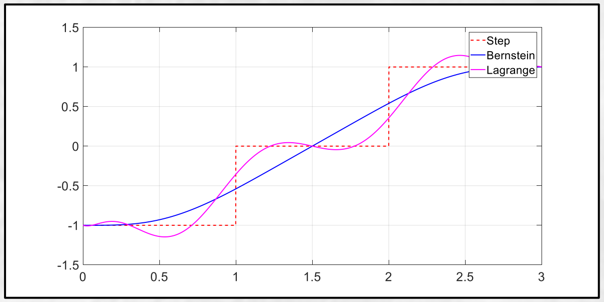




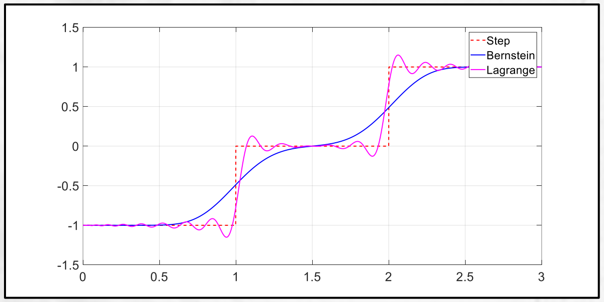
Approximating non-smooth functions

Bernstein approximations can be used to approximate piecewise continuous functions

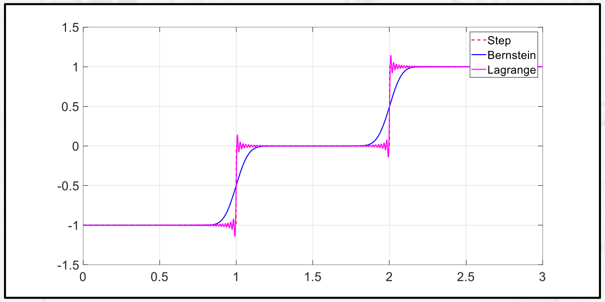
GIBBS PHENOMENON



N = 10



N = 50



N = 500

$$\lim_{n\uparrow\infty}\left(\max_{|x_0-x|\le\delta} f_n(x) - \min_{|x_0-x|\le\delta} f_n(x)\right) = C|f(x_0+0) - f(x_0-0)|.$$

$$C = \frac{2}{\pi}\int_0^{\pi}\left(\frac{\sin x}{x}\right)dx \approx 1.18.$$

Bernstein Approximation

$$\lim_{\delta\to 0}\lim_{n\to\infty}\left(\max_{|x_0-x|\le\delta} B_n f(x) - \min_{|x_0-x|\le\delta} B_n f(x)\right) = |f(x_0+0) - f(x_0-0)|.$$

Gzyl, Henryk, and José Luis Palacios. "On the approximation properties of Bernstein polynomials via probabilistic tools." *Boletın de la Asociación Matemática Venezolana* 10.1 (2003): 5-13.

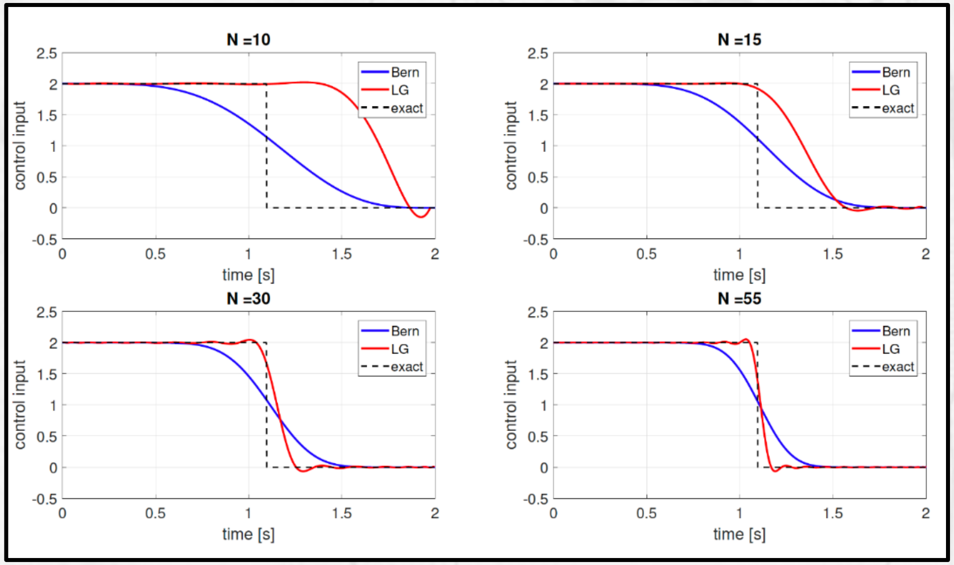
Approximating non-smooth functions

Minimize

$$I(y(t), u(t)) = \int_0^2 (3u(t) - 2y(t))dt\,,$$

subject to

$$\begin{aligned} &\dot{y}(t) = y(t) + u(t)\,, \quad \forall t \in [0,2],\\ &y(0) = 4\,,\\ &y(2) = 39.392\,,\\ &0 \le u(t) \le 2 \quad \forall t \in [0,2]\,. \end{aligned}$$



Optimal controller

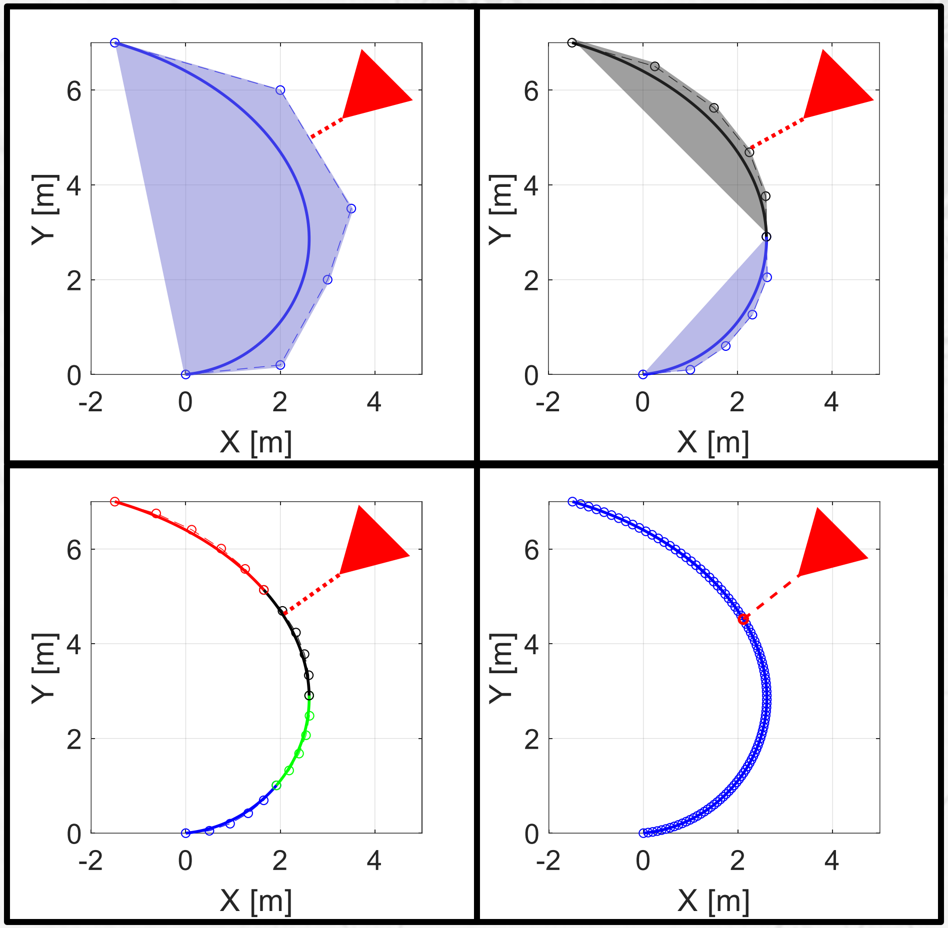
$$u^*(t) = \begin{cases} 2 & 0 \le t \le 1.096 \\ 0 & 1.096 \le t \le 2. \end{cases}$$

Gzyl, Henryk, and José Luis Palacios. "On the approximation properties of Bernstein polynomials via probabilistic tools." *Boletın de la Asociación Matemática Venezolana* 10.1 (2003): 5-13.

Bernstein polynomials properties

- Convex hull
  - A Bézier curve is contained within the convex hull defined by its control points
  - GJK algorithm computes distance between convex hulls (curve and obstacle)
- de Casteljau algorithm
  - Subdivides Bézier curve in multiple Bézier curves
- Distance between 2 curves, min/max velocity, acceleration, etc.
- Computational efficient algorithm for the computation of:

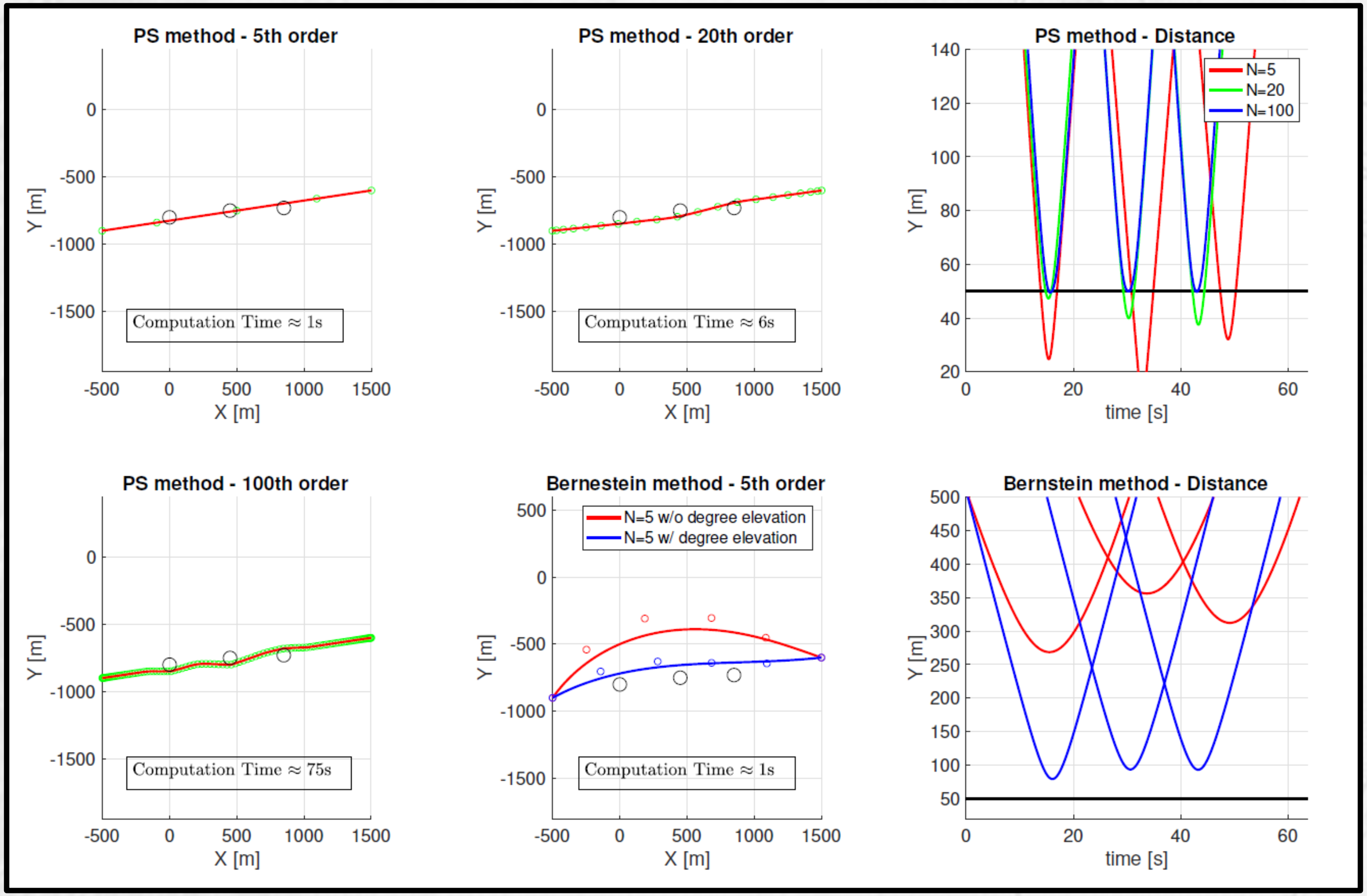
  Degree elevation; extrema of a Bernstein polynomial; minimum distance between Bernstein polynomials; penetration distance between Bernstein polynomial and convex shapes; see **BeBOT** at https://github.com/caslabuiowa/



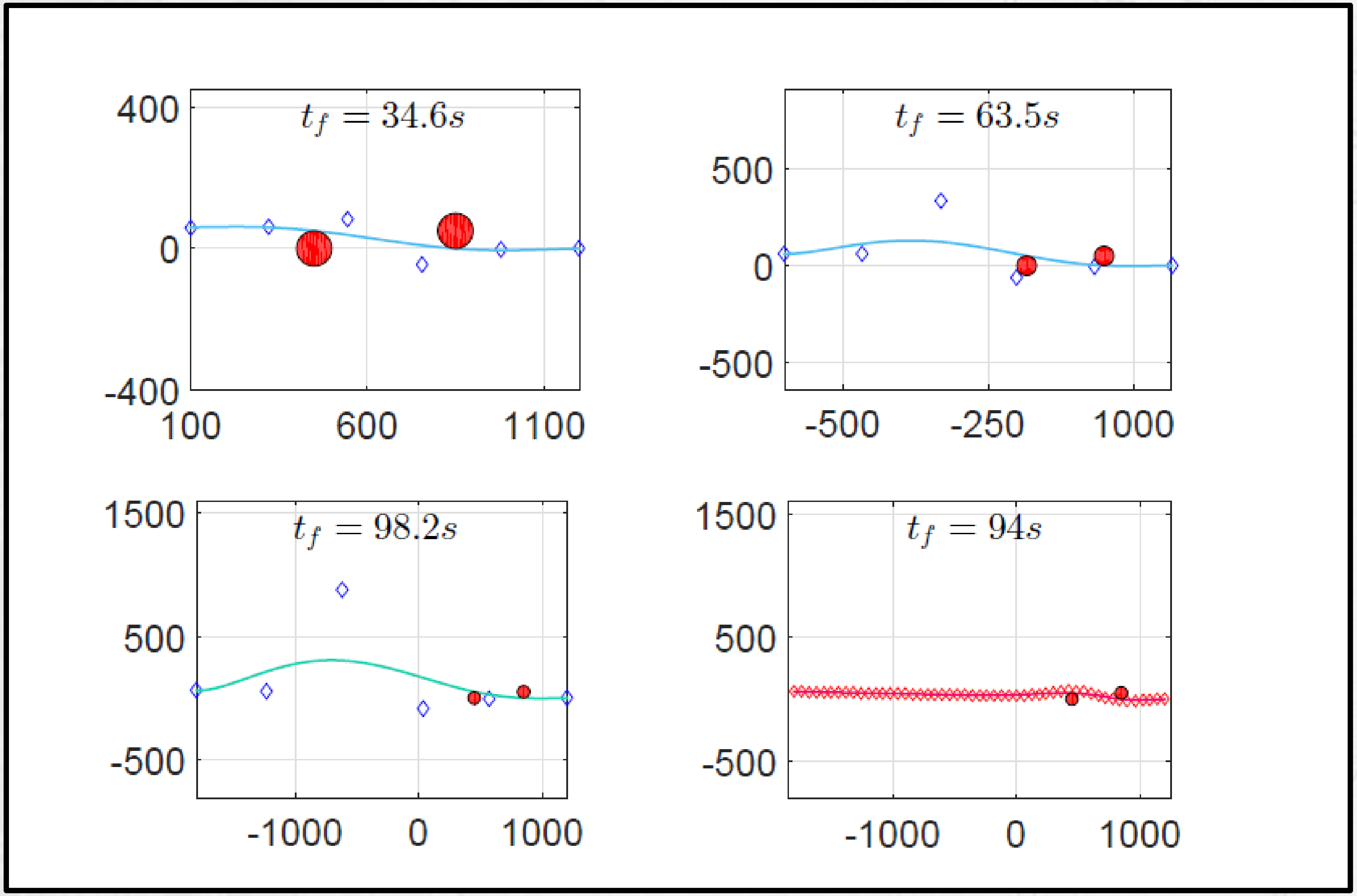
Collision avoidance is guaranteed even for **low-order** approximations
safe and computationally efficient; scalability

Kielas-Jensen, C. and Cichella, V., "BeBOT: A Python toolkin for motion planning using Bernstein polynomials", 2019, IEEE/RSJ International Conference on Intelligent Robots and Systems

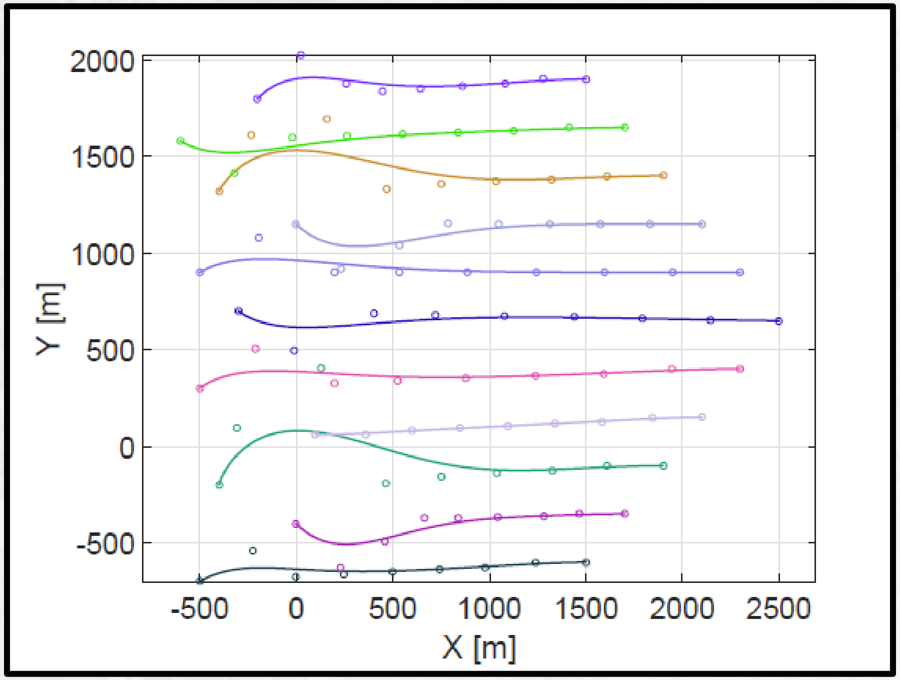
Numerical Results I

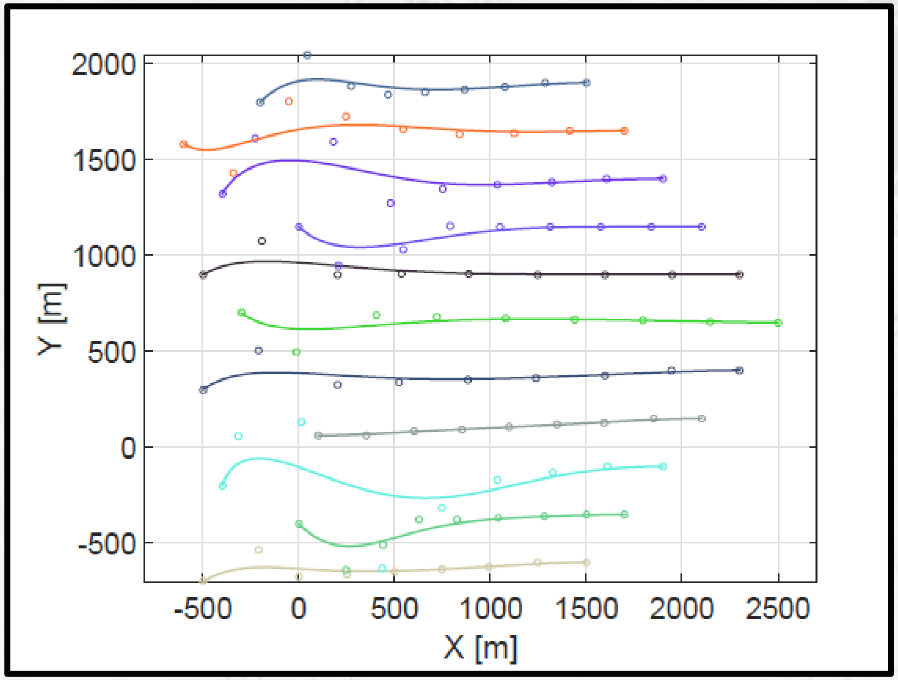


Numerical Results II



Numerical Results III



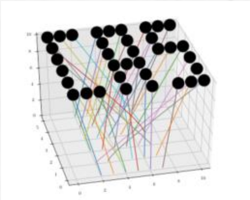


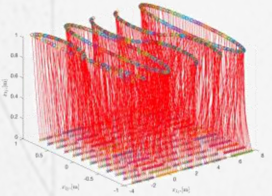
Temporal separation

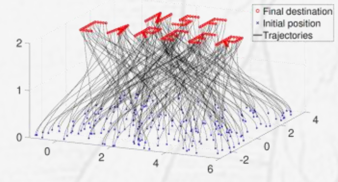
- Bernstein: <u>55 constraints</u>
- Pseudospectral: 550 constraints (<u>55*N</u>)

Spatial separation

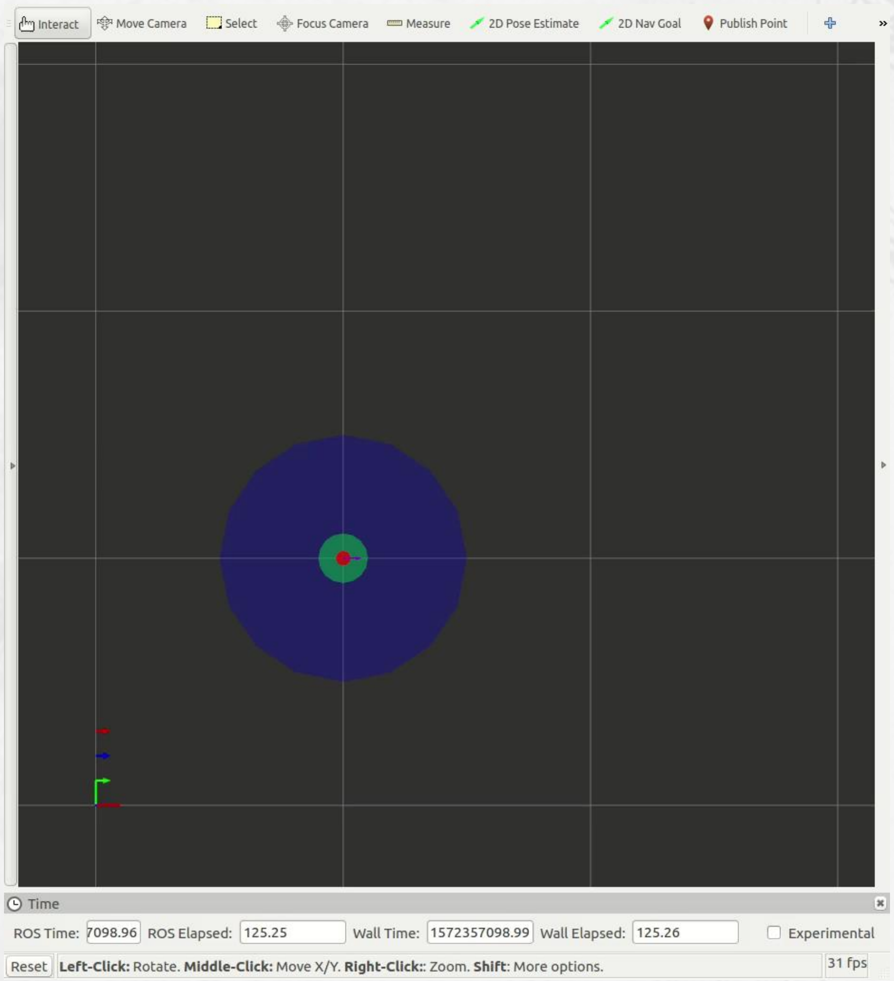
- Bernstein: <u>55 constraints</u>
- Pseudospectral: 5500 constraints (<u>55*N^2</u>)





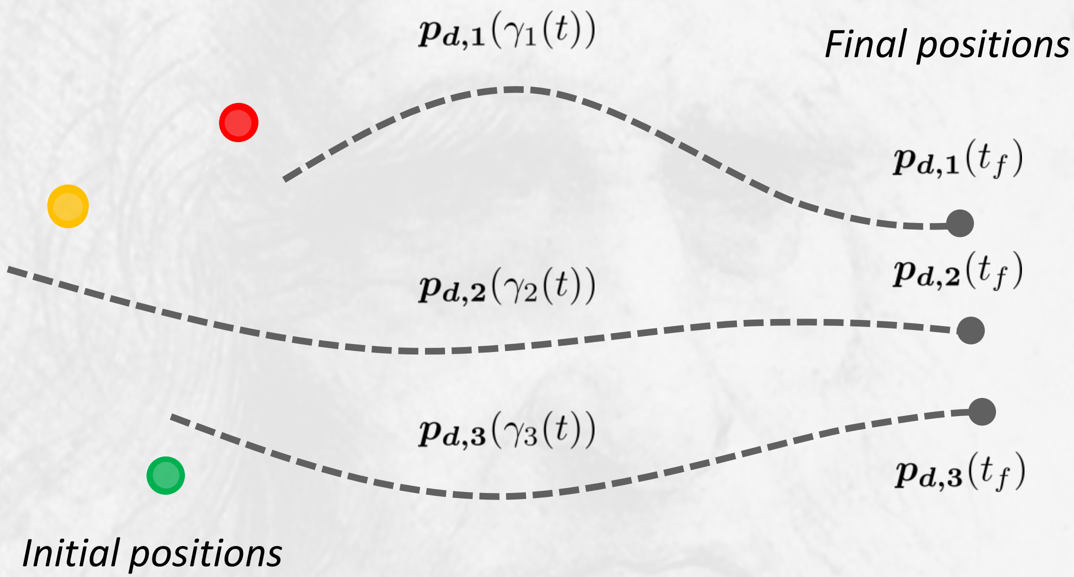


Numerical Results



Persistent monitoring of a target with unknown dynamics
Real-time optimal motion planning

COORDINATION OBJECTIVE



Simultaneous arrival

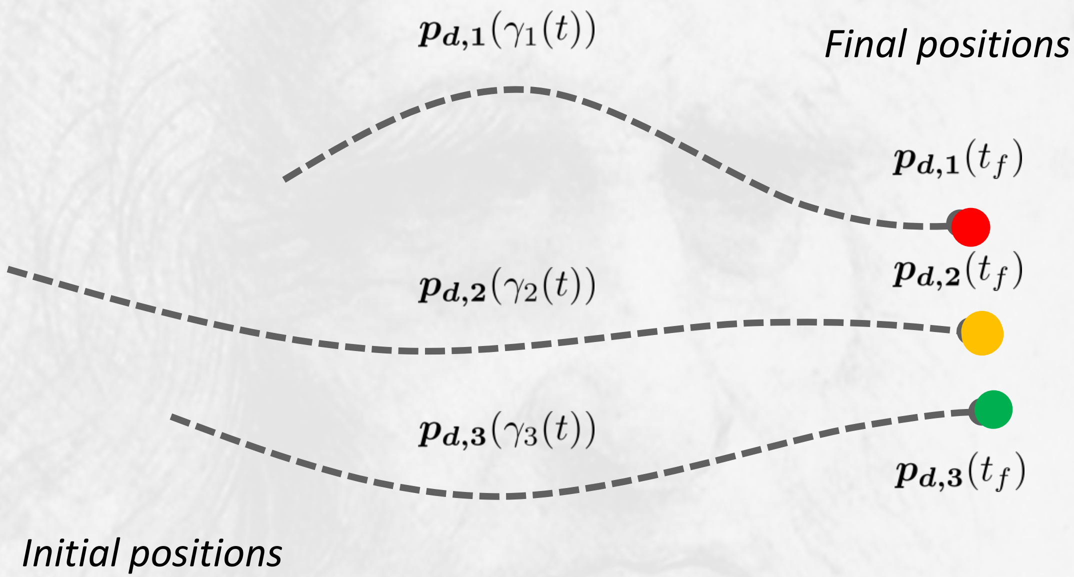
but…

Absolute time is <u>not</u> a priority

Consensus problem: reach an *agreement* on some distributed variables of interest *(coordination states)*

$$\gamma_i(t) - \gamma_j(t) \xrightarrow{t\to\infty} 0\,, \quad \forall\, i,j = 1,\dots,n$$

COORDINATION OBJECTIVE



Simultaneous arrival

but…

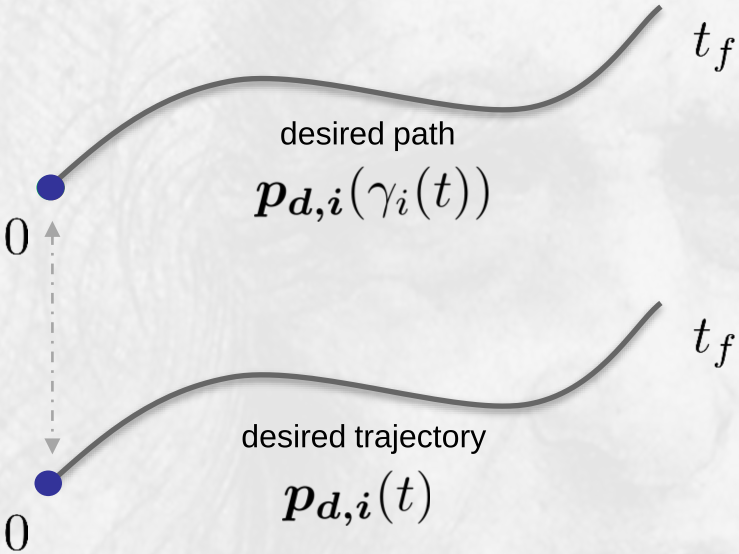
Absolute time is <u>not</u> a priority

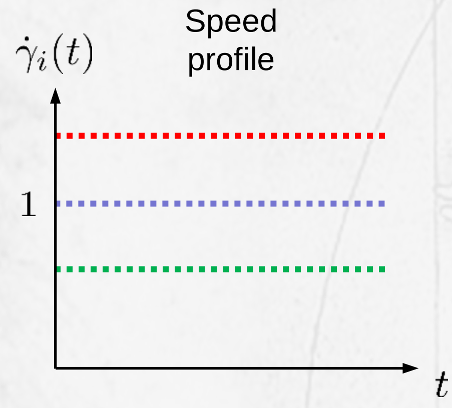
Consensus problem: reach an *agreement* on some distributed variables of interest *(coordination states)*

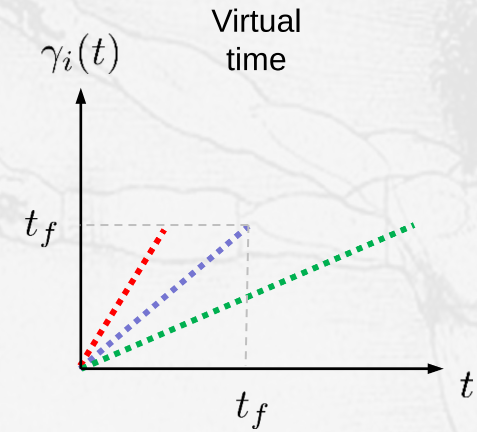
$$\gamma_i(t) - \gamma_j(t) \overset{t\to\infty}{\longrightarrow} 0\,, \quad \forall\, i,j = 1,\dots,n$$
$$\dot{\gamma}_i(t) \overset{t\to\infty}{\longrightarrow} r(t)\,, \quad \forall\, i = 1,\dots,n$$

Synchronize in both 'position' and 'speed'

COORDINATION OBJECTIVE







Consensus problem: reach an *agreement* on some distributed variables of interest *(coordination states)*

$$\gamma_i(t) - \gamma_j(t) \xrightarrow{t\to\infty} 0\,, \quad \forall\, i,j = 1,\ldots,n$$

$$\dot{\gamma}_i(t) \xrightarrow{t\to\infty} r(t)\,, \quad \forall\, i = 1,\ldots,n$$

Synchronize in both 'position' and 'speed'

COORDINATION CONTROL LAW

- Distributed control law for group coordination:

$$\ddot{\gamma}_1(t) = -b\left(\dot{\gamma}_1(t) - \dot{\gamma}_d(t)\right) - a \sum_{j \in \mathcal{N}_1} \left(\gamma_1(t) - \gamma_j(t)\right),$$

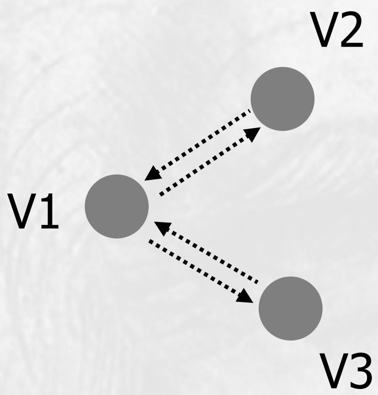
$$\ddot{\gamma}_i(t) = -b\left(\dot{\gamma}_i(t) - \chi_{I,i}(t)\right) - a \sum_{j \in \mathcal{N}_i} \left(\gamma_i(t) - \gamma_j(t)\right),$$

$$\dot{\chi}_{I,i}(t) = -k \sum_{j \in \mathcal{N}_i} \left(\gamma_i(t) - \gamma_j(t)\right), \qquad \forall i \in \{2, \ldots, n\},$$

Each vehicle exchanges only its **coordination state** with its neighbors

Under which assumptions on the communication network this control law guarantees that the coordination objective is attained?

COMMUNICATION NETWORK



Laplacian Matrix $\boldsymbol{L} = \begin{bmatrix} 2 & -1 & -1 \\ -1 & 1 & 0 \\ -1 & 0 & 1 \end{bmatrix}$

The graph is connected if

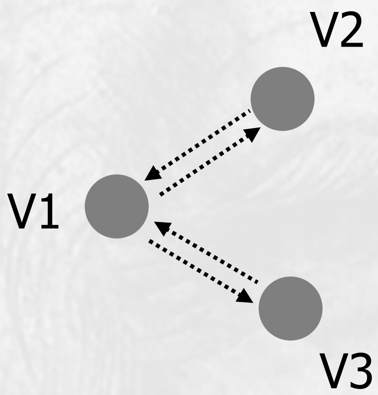
$$\text{rank}\boldsymbol{L} = n - 1 = 2$$

V1 receives info from *neighbours* V2 and V3

V2 receives info from *neighbour* V1

V3 receives info from *neighbour* V1

COMMUNICATION NETWORK



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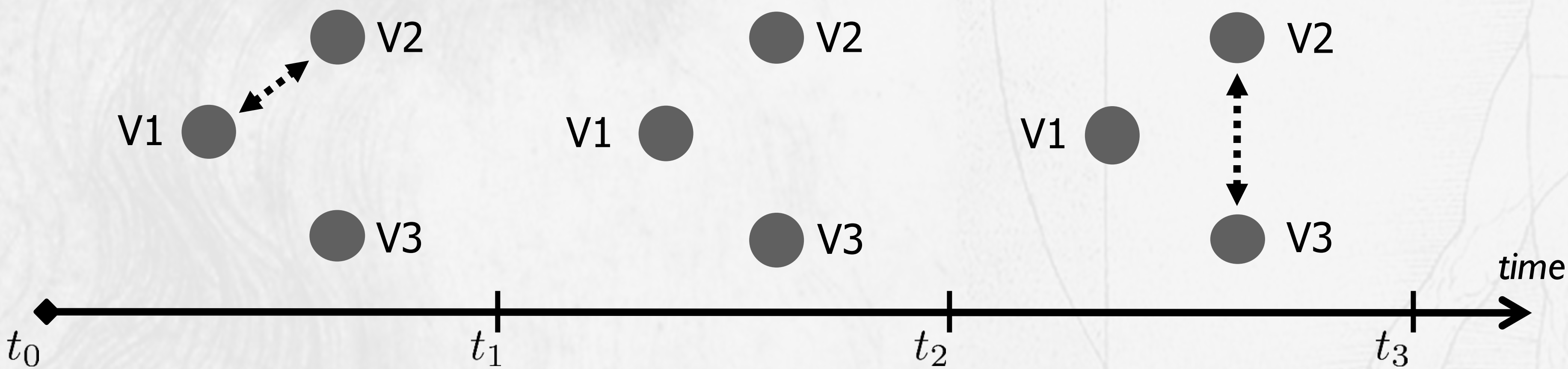
The graph is connected if

$$\text{rank}\boldsymbol{L} = n - 1 = 2$$

Graph connected in the mean



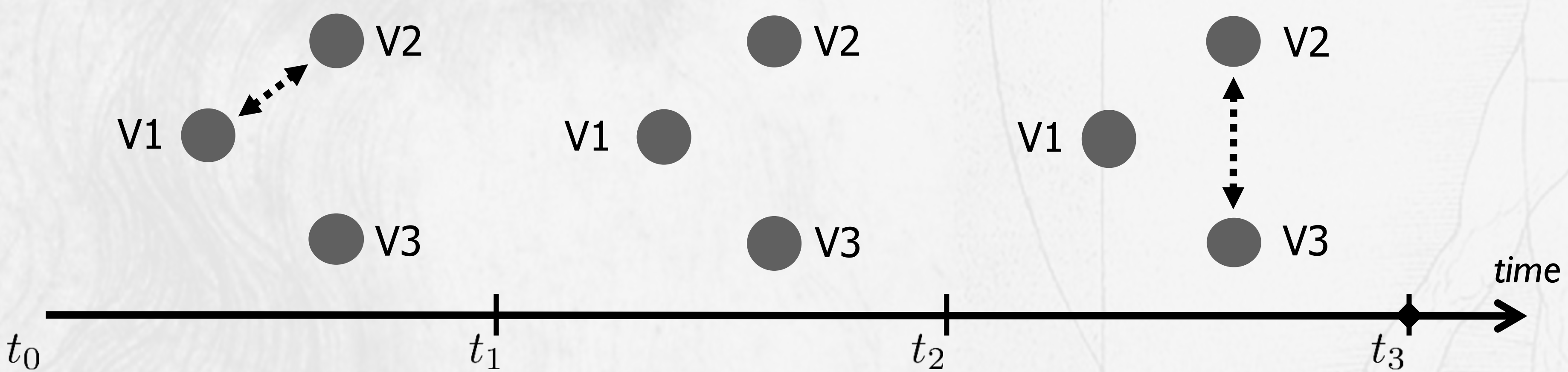
$$\boldsymbol{L}_T = \boldsymbol{L}_{[t_0,t_1]} + \boldsymbol{L}_{[t_1,t_2]} + \boldsymbol{L}_{[t_2,t_3]}\,, \qquad \text{rank}\boldsymbol{L}_T = n - 1 = 2$$

COMMUNICATION NETWORK

$$\frac{1}{n}\frac{1}{T}\int_{t}^{t+T} \boldsymbol{Q}\,\boldsymbol{L}(\tau)\,\boldsymbol{Q}^\top d\tau \geq \mu\,\mathbb{I}_{\boldsymbol{n-1}}\,, \qquad \forall\, t \geq 0, \qquad \boldsymbol{Q}\boldsymbol{1_n} = 0$$

Network connected in an integral sense, not pointwise in time (Arcak 2007)

Parameters μ and T characterize the **QoS of the network**



$$\boldsymbol{L}_T = \boldsymbol{L}_{[t_0,t_1]} + \boldsymbol{L}_{[t_1,t_2]} + \boldsymbol{L}_{[t_2,t_3]}\,, \qquad \mathrm{rank}\boldsymbol{L}_T = n-1 = 2$$

COORDINATION: MAIN RESULT

- Assume **network connectivity** satisfies

$$\frac{1}{n}\frac{1}{T}\int_{t}^{t+T} \boldsymbol{Q}\,\boldsymbol{L}(\tau)\,\boldsymbol{Q}^\top d\tau \geq \mu\,\mathbb{I}_{\boldsymbol{n-1}}\,, \qquad \forall\; t \geq 0$$

- The coordination states $\boldsymbol{x_{cd,i}}(t) = \left[\sum_{j=1}^{n}(\gamma_i(t)-\gamma_j(t))\,,\quad \dot{\gamma}_i(t)-1\right]$ *satisfy*

$$\|\boldsymbol{x_{cd,i}}(t)\| \leq \kappa_1\|\boldsymbol{x_{cd,i}}(0)\|e^{-\lambda_{cd}t} + \kappa_2\sup_{t\geq 0}\|\boldsymbol{p_{d,i}}(\gamma_i(t)) - \boldsymbol{p_i}(t)\|$$

- For ideal performance of the autopilot *the coordination states* ***exponentially*** *with rate of convergence*

$$\lambda_{cd} \geq \bar{\lambda}_{cd} \triangleq \frac{a}{b}\frac{n\mu}{T\left(1+\frac{a}{b}nT\right)^2}$$

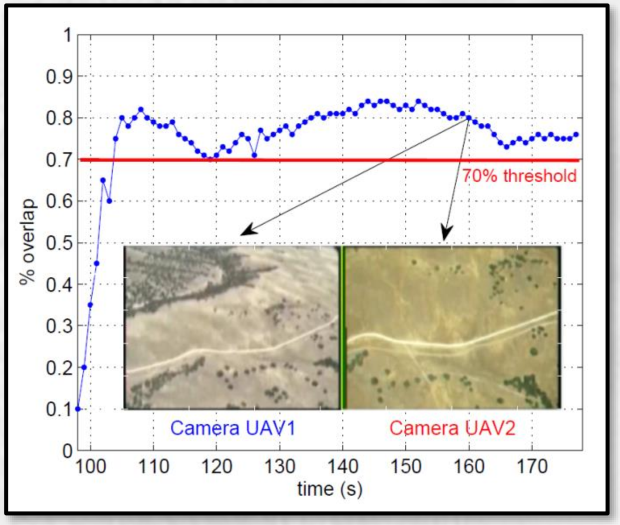
- Moreover, $\boldsymbol{p_{d,i}}(\gamma_i(t))$ is feasible.

AUTOPILOT PERFORMANCE

QoS of the communication network

V. Cichella, I. Kaminer, V. Dobrokhodov, E. Xargay, R. Choe, N. Hovakimyan, A. P. Aguiar, and A. M. Pascoal. "Cooperative path following of multiple multirotors over time-varying networks." *IEEE Transactions on Automation Science and Engineering* 12, no. 3 (2015): 945-957.

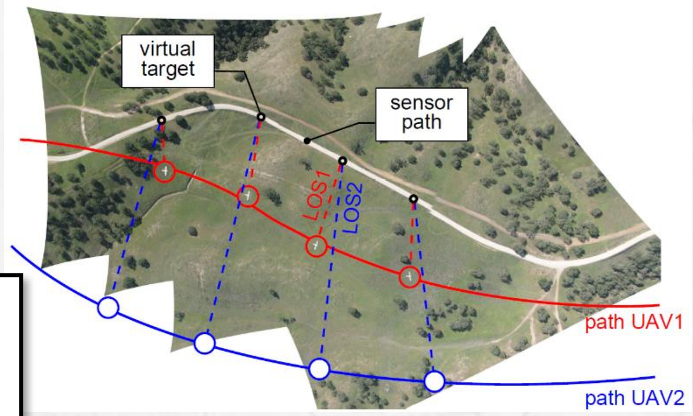
RESULTS



Cooperation** ensures satisfactory overlap of the field-of-view footprints of the sensors, increasing the probability of **target detection

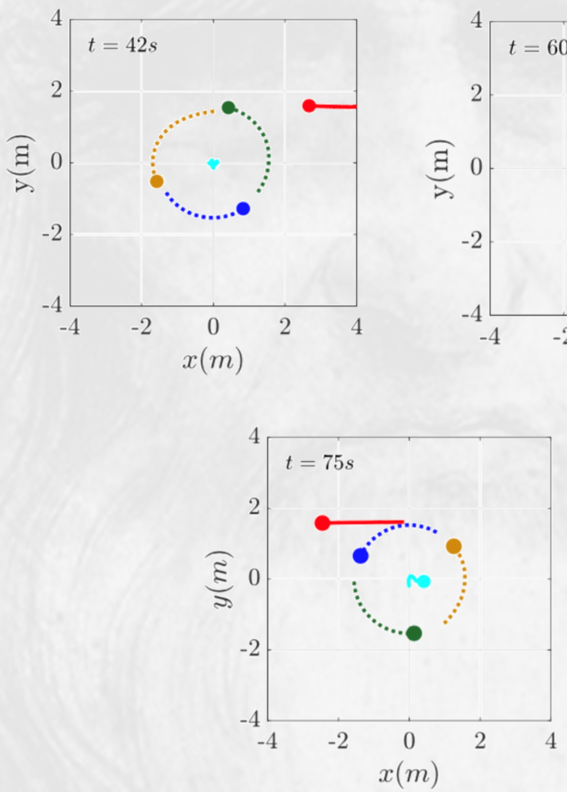
UAV 1

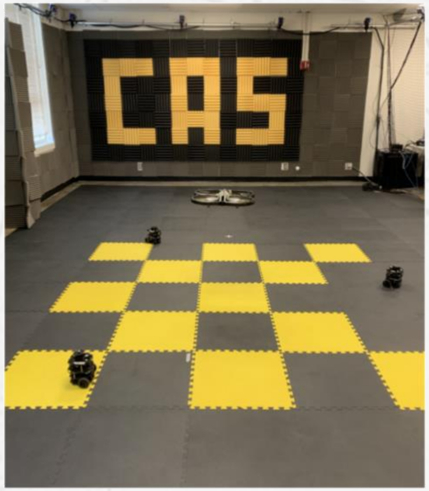
UAV 2

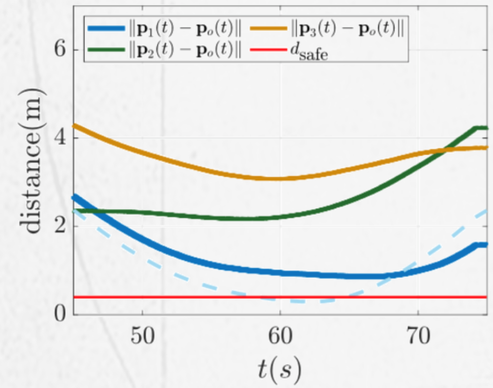


Mosaic of 4 consecutive high-resolution images

RESULTS





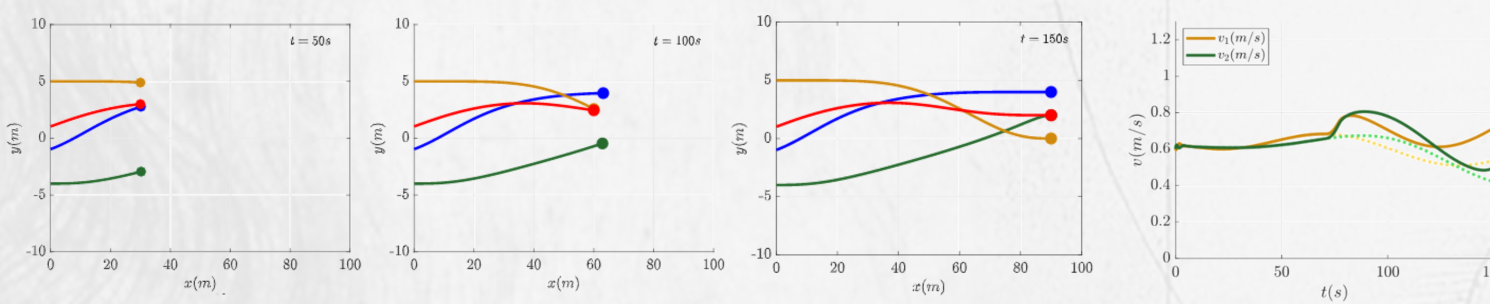


Multiple drones and ground robots performing cooperative collision avoidance maneuvers

RESULTS

Multiple drones and ground robots performing cooperative collision avoidance maneuvers

RESULTS



Multiple UAVs performing cooperative target tracking maneuvers

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Thank you

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