NOVEL GUIDANCE AND NONLINEAR CONTROL SOLUTIONS IN PRECISION GUIDED PROJECTILES

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OUTLINE:

I. Introduction to Precision Guided Projectiles

- SGP 5” Projectile Flight Test Video
- Flight Profile Timeline Quiz
- Guidance, Navigation, Control Block Diagram
- Guidance, Navigation and Control Issues

II. Guidance Law – GENEX

III. Nonlinear Robust Control Law Design

- Background – Dynamic Inversion, Feedback Linearization, Back-stepping, Robust Recursive

IV. Nonlinear Recursive Pitch-Yaw Autopilot Design for a Guided Projectile

- Nonlinear Recursive Pitch-Yaw Controller Design
- Aerodynamic Functions for Hypothetical Projectile
- Pitch-Yaw Controller Equations
- Mass Properties for Hypothetical Projectile
- MATLAB Simulation Results for Hypothetical Pitch-Yaw Projectile Controller

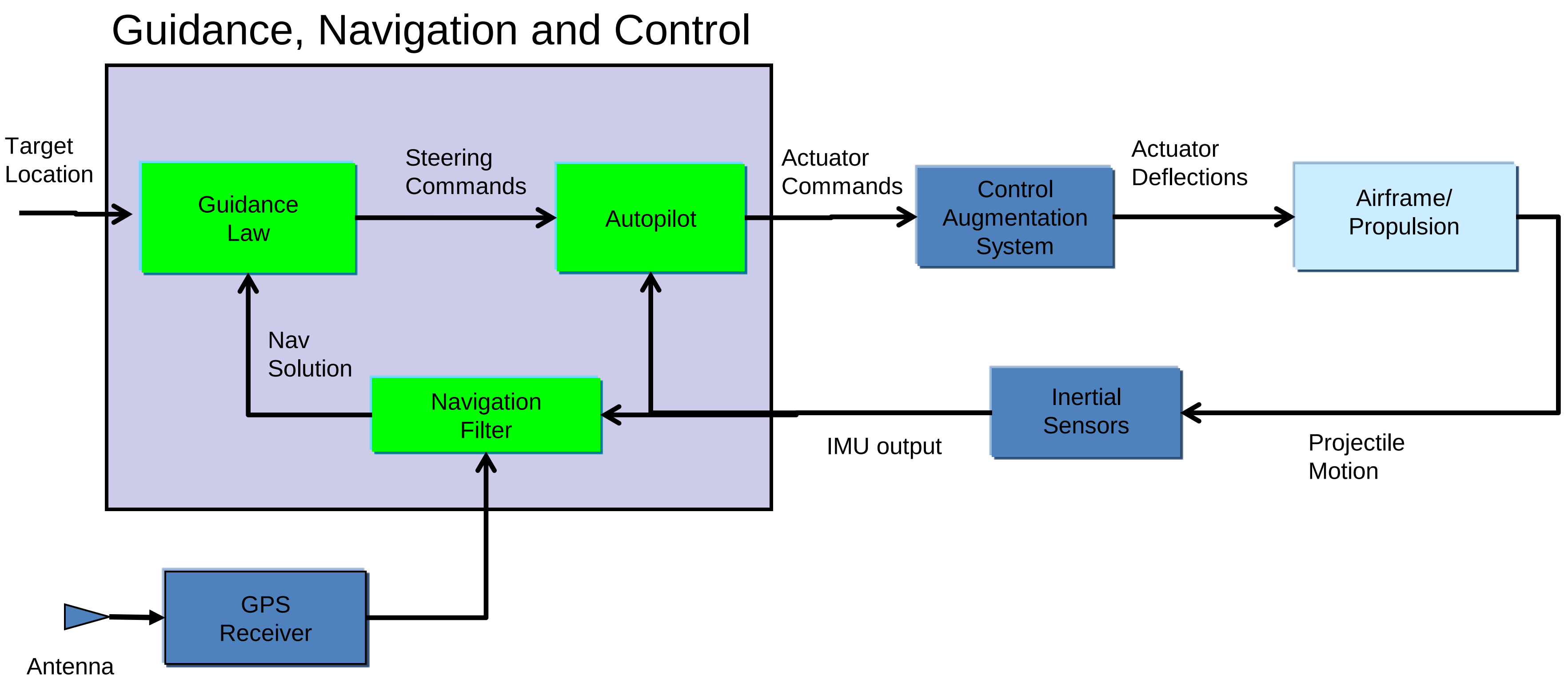
V. Conclusions

5-inch Multi Service Standard Guided Projectile

- Video Courtesy of BAE Systems

https://www.bing.com/videos/search?q=Navy+5+inch+guided+projectile+video&view=detail&mid=E67446D3A21E0F0E9D7FE67446D3A21E0F0E9D7F&FORM=VIRE

SYSTEM BLOCK DIAGRAM



GUIDANCE ISSUES:

Minimum Control Effort Optimal Control Problem with:

- Final Position (Miss Distance) Constraint
- Terminal Angle of Fall (Velocity Unit Vector) Constraint
- Control Action Constraint (Normal to Velocity Vector)
- Nonlinear Dynamics (True Optimal Solution is TPBV Problem)
- May be Final Time (Time on Target) Constraint

Several Well Known Sub-Optimal Solutions

- Modifications to Biased Proportional Nav
- Explicit Guidance
- Generalized Explicit Guidance (GENEX)
- Not well suited to Guided Projectiles of “Limited Maneuver Capability”

NAVIGATION ISSUES:

- Electronics Must Be Densely Packaged and Low Cost!
- Inertial Sensors Must Survive High “G” Gun Launch Environment (~10,000 g’s)
- GPS Must Acquire Quickly, In-Flight, On A Rapidly Moving, Spinning Projectile
- Navigation Filter Must Initialize and Self-Orient In Mid-Flight
- What to Do if GPS is Unavailable

CONTROL ISSUES:

- Ballistic Portion of Flight Must Avoid Roll Resonance Issues
- Airframe Control Capture at Canard Deployment
  - Airframe May Become Unstable at Canard Deploy
  - IMU Sensors May Be Saturated at Canard Deploy
    - High Roll Rate
    - Deployment Shock
- Large Flight Envelope of Mach and Dynamic Pressure
- Rapidly Changing Flight Envelope
- Winds and Variations in Atmospheric Properties Cause Significant Uncertainties in Gain Scheduled Parameters (No Air Data Probes)
- Highly Nonlinear and Uncertain Aerodynamics
  - Canard "Vortex Shedding" Interactions with Tails
  - Uncertain Tail "Clocking" Relative to Canards

EXPLICIT GUIDANCE

Solution to a Time Varying Linear Optimal Control Problem of the form:

$$\text{minimize} \quad J = \int_0^{t_f} \frac{u^2}{2} dt$$

Subject to the following state constraints:

$$\dot{x}_1 = x_2 - Tu$$
$$\dot{x}_2 = -u$$

Which yields the optimal control:

$$u = \frac{1}{T^2}\left[6x_1 - 2T x_2\right]$$

Note – we often augment this with an acceleration term to include known effects of gravity and drag.

Where we define:

$$x_1 = y_f - y - \dot{y}T = \text{position error}$$
$$x_2 = \dot{y}_f - \dot{y} = \text{velocity error}$$
$$T = t_f - t = \text{time-to-go}$$
$$u = \text{acceleration control}$$

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GENERALIZED EXPLICIT GUIDANCE (GENEX)

"Generalized Vector Explicit Guidance", Ernest J. Ohlmeyer and Craig A. Phillips, AIAA *Journal of Guidance, Control and Dynamics*, Vol. 29, No. 2, March-April 2006

Solution to a Time Varying Linear Optimal Control Problem of the form:

$$\text{minimize} \quad J = \int_0^{t_f} \frac{u^2}{2T^n} dt$$

Family of functions, parameterized by scalar n
Higher n allows greater penalty weight on control usage as T → 0

Subject to the following state constraints:

$$\dot{x}_1 = x_2 - Tu$$
$$\dot{x}_2 = -u$$

Where we define:

$$x_1 = y_f - y - \dot{y}T = \text{position error}$$
$$x_2 = \dot{y}_f - \dot{y} = \text{velocity error}$$
$$T = t_f - t = \text{time-to-go}$$
$$u = \text{acceleration control}$$

Which yields the optimal control:

$$u = \frac{1}{T^2}\left[k_1 x_1 + k_2 x_2 T\right]$$

where:

$$k_1 = (n+2)(n+3)$$
$$k_2 = -(n+1)(n+2)$$

Note that n = 0 results in k_1 = 6 and k_2 = -2 reducing to the standard explicit guidance gains

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GENEX with Gravity Term (NEW):

Note on Updated Derivation of Generalize Explicit Guidance to Include Gravity Acceleration Term from Ernie Ohlmeyer, May 2016

Solution to a Time Varying Linear Optimal Control Problem of the form:

$$\text{minimize} \quad J = \int_0^{t_f} \frac{u^2}{2T^n} dt$$

Family of functions, parameterized by scalar n
Higher n allows greater penalty weight on control usage as T → 0

Subject to the following state constraints:

$$\dot{x}_1 = x_2 - Tu$$

$$\dot{x}_2 = -u$$

Which yields the optimal control:

$$u = \frac{1}{T^2}\left[k_1 x_1 + k_2 x_2 T\right] + k_3 g$$

$$\boxed{\begin{aligned} \text{where:} \quad k_1 &= (n+2)(n+3) \\ k_2 &= -(n+1)(n+2) \\ k_3 &= \frac{(n+2)(n-1)}{2} \end{aligned}}$$

Setting $k_3 = 0$ recovers original GENEX

Where we define:

$$x_1 = y_f - y - \dot{y}T - \frac{1}{2}gT^2 = \text{position error}$$

$$x_2 = \dot{y}_f - \dot{y} - gT = \text{velocity error}$$

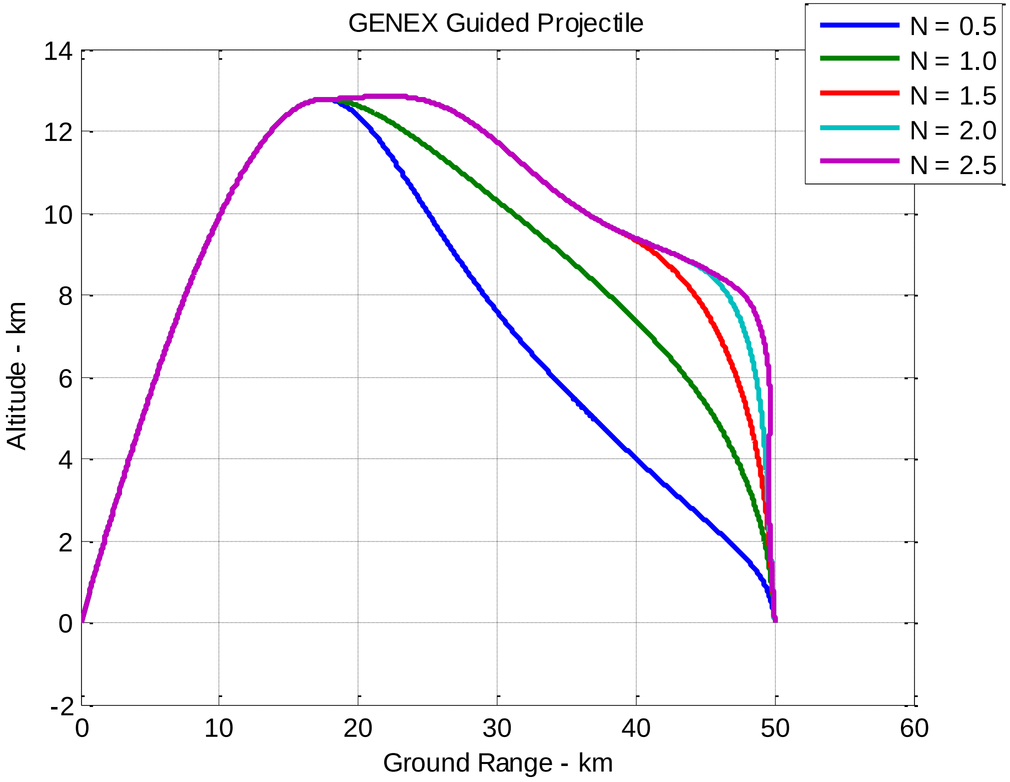
$$T = t_f - t = \text{time-to-go}$$

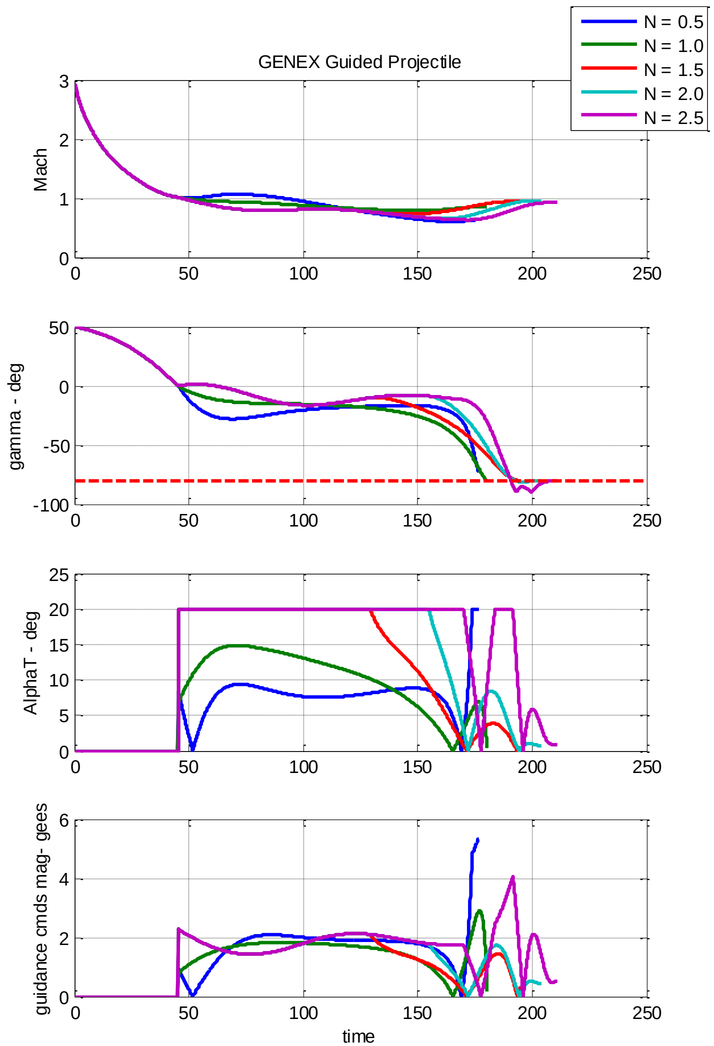
$$u = \text{acceleration control}$$

$$g = \text{acceleration of gravity}$$

EXAMPLE TRAJECTORIES USING GENEX

GENEX Guidance from Apogee
V0 = 1000 m/s
Launch Elevation = 50 deg
Final Gamma = -80 deg
Target Range = 50 km

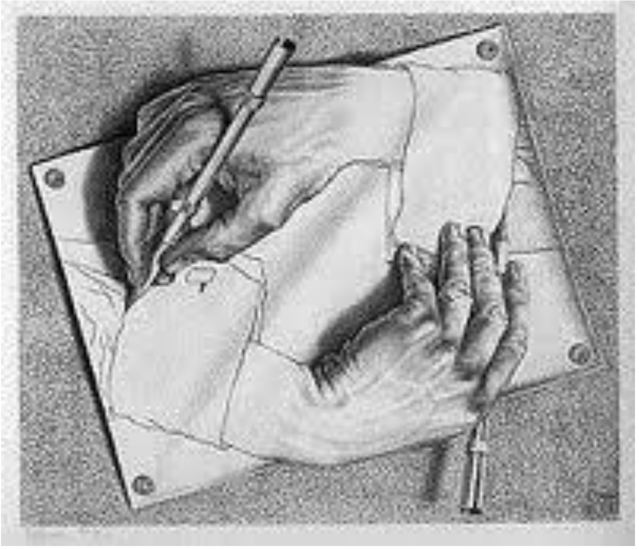




RECURSIVE CONTROL DESIGN:

"Recursivist" – one who views the world as a giant opportunity to apply the rigorous methods of *recursive* nonlinear control design, one layer at a time!

See also *"Integrator Backsteppinger","Dynamic Inversionist" and "Feedback Linearizationist"*



DYNAMIC INVERSION

Given a nonlinear system "affine" w.r.t. the control input:

$$\dot{x} = f(x) + g(x)u$$

$$x \in \Re^n, \quad u \in \Re^n$$

If g(x) is invertible the control:

$$u = g^{-1}(x)[\dot{x}_d - f(x)]$$

Will cause the system to track the desired dynamics :

$$\dot{x} = \dot{x}_d$$

Note that we are not actually inverting the dynamics of the entire system, only the algebraic input connection matrix!

DYNAMIC INVERSION

The primary restrictions on Dynamic Inversion are:

1. The system must be scalar or *square* – that is it must have the same number of inputs as states, and
2. The matrix g(x) must be non-singular *over the entire region of interest.*

Invertibility of g(x) also insures stability of the zero dynamics, so that the system is minimum phase.

In fact, it is a stronger condition, which guarantees that the system can be decoupled into *n* independently controllable subsystems by the *n* control inputs.

This condition is somewhat rare in real systems!

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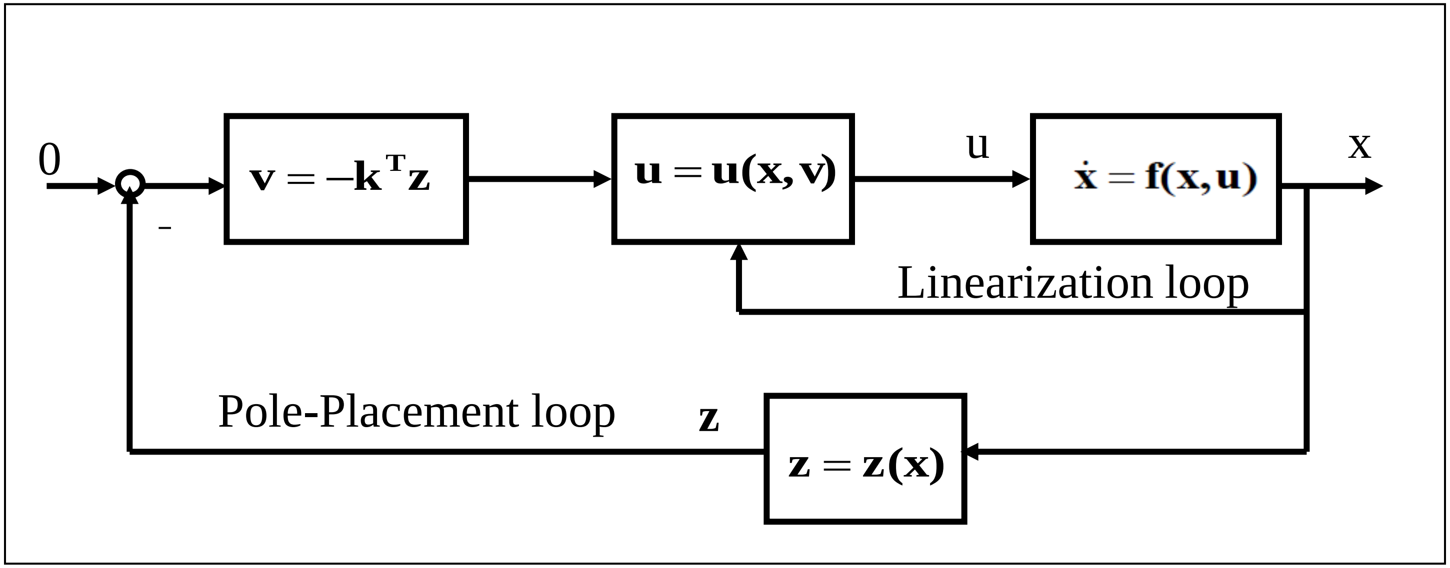
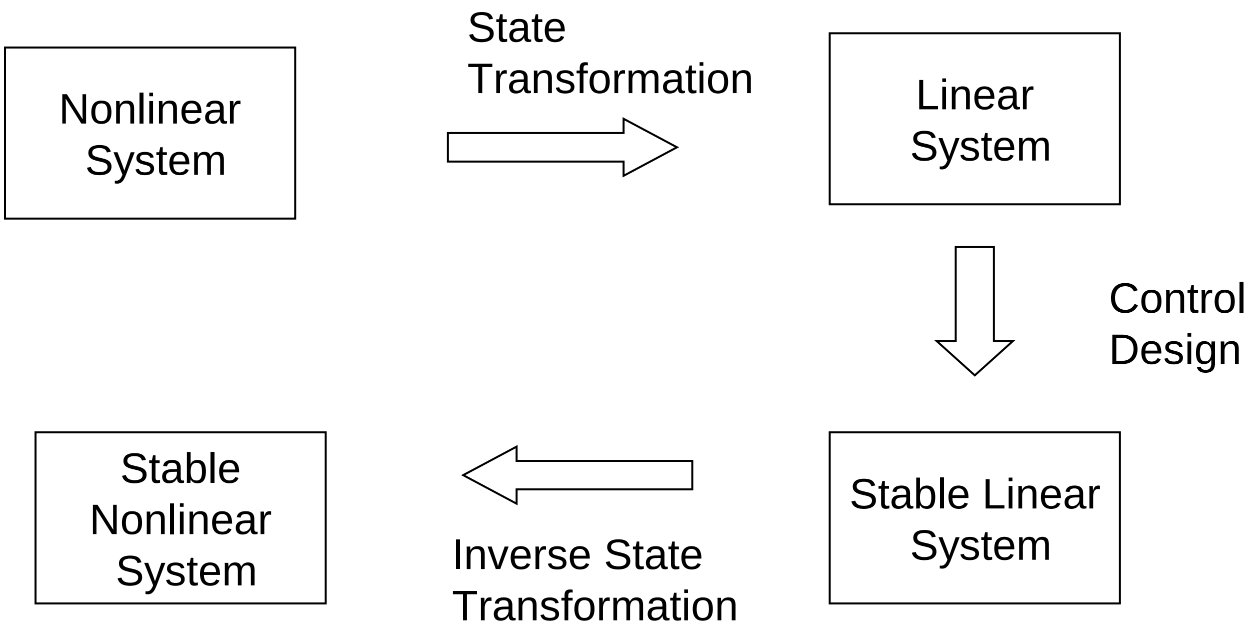
FEEDBACK LINEARIZATION

- Not Conventional (Jacobi) Linearization
- Systematic Method for finding a state transformation that maps the system to an "equivalent" linear system.
- Design Control for the Linear System
- Map the Control back to the original nonlinear system.



FEEDBACK LINEARIZATION

Feedback Linearization involves a transformation of state variables in order to make the control design and stability analysis of the system easier.



But, in order to guarantee "equivalence" of dynamics and transference of stability properties between the linear and nonlinear systems, we must rely on concepts from set theory, and differential geometry.

FEEDBACK LINEARIZATION - COMMENTS

Input-Output Feedback Linearization is more desirable for the control of missiles and aircraft, in which output tracking (command following) is of interest.

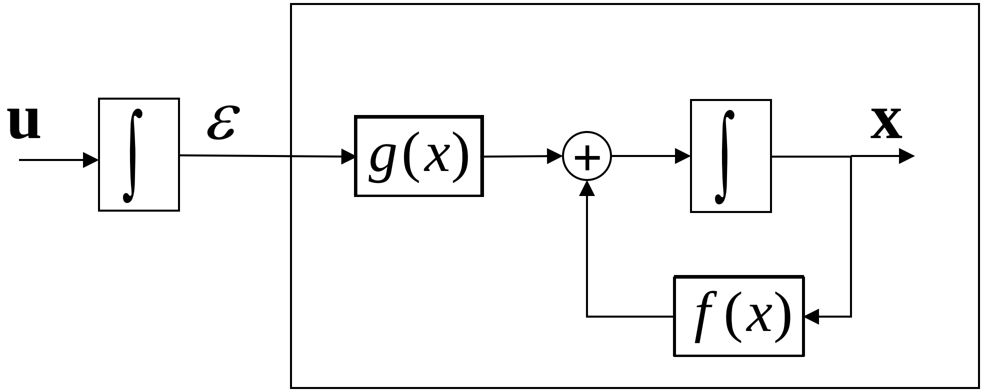
Exact Output Tracking is not possible for non-minimum phase systems, which are equivalent to systems with less than full relative degree ($r < n$), and unstable zero dynamics.

Tail Control of Aircraft and Missiles exhibits minimum phase response in alpha and beta, but non-minimum phase characteristics in normal accelerations.

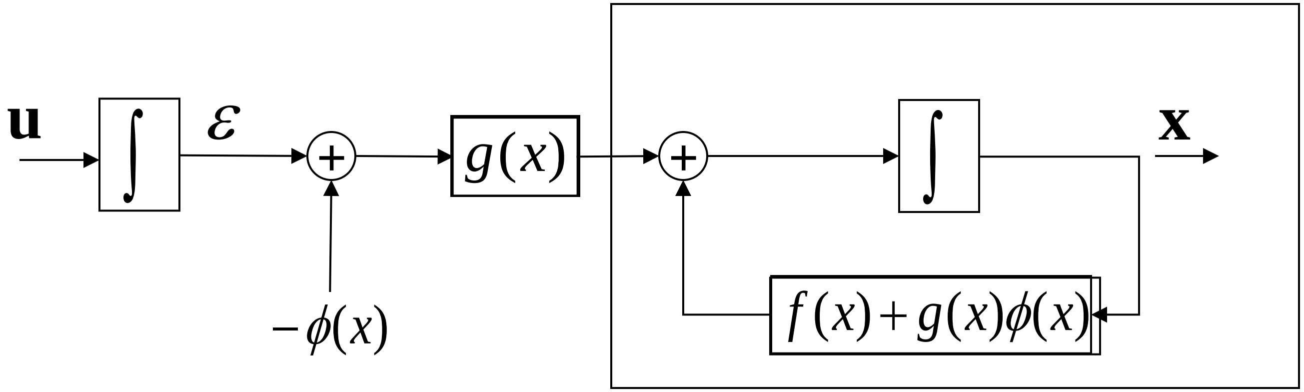
The machinery of State Feedback Linearization could be used to find an alternative output for non-minimum phase acceleration tracking – however the transformations are fairly intractable .

INTEGRATOR BACKSTEPPING

Given:

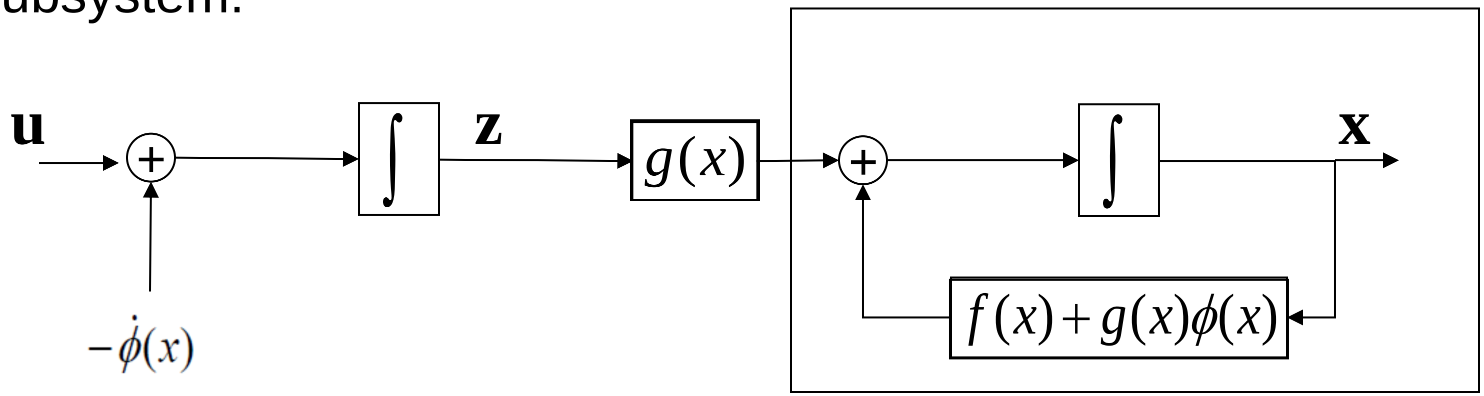


Design a stabilizing "fictitious: control $\phi(x)$ for the output subsystem:



INTEGRATOR BACKSTEPPING

Then “backstep” that control through the integrator of the previous subsystem:



It can be seen that a change of variable $z=\varepsilon-\phi(x)$ is equivalent to adding zero to the original subsystem:

$$\dot{x}=f(x)+g(x)\big(\varepsilon-\phi(x)+\phi(x)\big)$$

Resulting in an equivalent subsystem:

$$\dot{x}=f(x)+g(x)\phi(x)+g(x)z$$

$$\dot{z}=u-\dot{\phi}(x)$$

RECURSIVE DESIGN

By ***recursive*** application of Integrator Backstepping, we can extend the results to higher order systems provided they have the ***strict feedback cascaded*** form:

$$\boxed{\begin{aligned}
\dot{x}_1 &= f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + g_1(x_1, x_2)x_3 \\
&\vdots \\
\dot{x}_n &= f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u
\end{aligned}}$$

At each step we define a new state variable:

$$\begin{aligned}
z_1 &= x_1 - x_1^d \\
z_2 &= \dot{z}_1 - \dot{z}_1^d \\
z_3 &= \dot{z}_2 - \dot{z}_2^d \\
&\vdots \\
z_n &= \dot{z}_{n-1} - \dot{z}_{n-1}^d
\end{aligned}$$

Until the control design equation "pops" out:

$$0 = \dot{z}_n - \dot{z}_n^d \Rightarrow u = \ldots$$

RECURSIVE DESIGN

Also at each step, we recursively accumulate the terms of the Lyapunov function:

$$V = \frac{1}{2} z_1^2 + \frac{1}{2} z_2^2 + \Lambda + \frac{1}{2} z_n^2$$

For which it can be shown: $\dot{V}(z) < 0$

Advantages of Recursive Backstepping Design over Feedback Linearization

1. Transformation is simple (no PDE’s to solve)
2. No need to cancel “beneficial” nonlinearities
3. No need to cancel “weak” nonlinearities
4. Can add robust stabilizing terms to overcome uncertainties

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ROBUST RECURSIVE DESIGN

Model the system with uncertainties:

$$\begin{aligned}
\dot{x}_1 &= f_1(x_1) + \Delta f_1(x_1) + g_1(x_1)x_2 \\
\dot{x}_2 &= f_2(x_1, x_2) + \Delta f_2(x_1, x_2) + g_1(x_1, x_2)x_3 \\
&\vdots \\
\dot{x}_n &= f_n(x_1, x_2, \ldots, x_n) + \Delta f_n(x_1, x_2, \ldots, x_n) + g_n(x_1, x_2, \ldots, x_n)u
\end{aligned}$$

Which meet the *Generalized Matching Conditions*

Bound the uncertainty that appears in the $\dot{z}_i$ equations:

$$\left\| \Delta f(z_i) \right\| \le \rho_i$$

Add a robust fictitious control term to the

Such as:

$$\begin{aligned}
z_{i+1} &= \dot{z}_i - \dot{z}_i^d + \nu_i \\
\nu_i &= \left(1 - \operatorname{sgn}(z_i) e^{-\sigma_i |z_i|}\right)\rho_i
\end{aligned}$$

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DYNAMIC RECURSIVE DESIGN

Dynamic Recursive Design extends the Recursive Process to systems which do not meet the strict feedback cascaded form:

$$\dot{x}_1 = f_1(x_1, x_2, \dots, x_n) + g_1(x_1, x_2, \dots, x_n)u$$
$$\dot{x}_2 = f_2(x_1, x_2, \dots, x_n) + g_2(x_1, x_2, \dots, x_n)u$$
$$\vdots$$
$$\dot{x}_n = f_n(x_1, x_2, \dots, x_n) + g_n(x_1, x_2, \dots, x_n)u$$

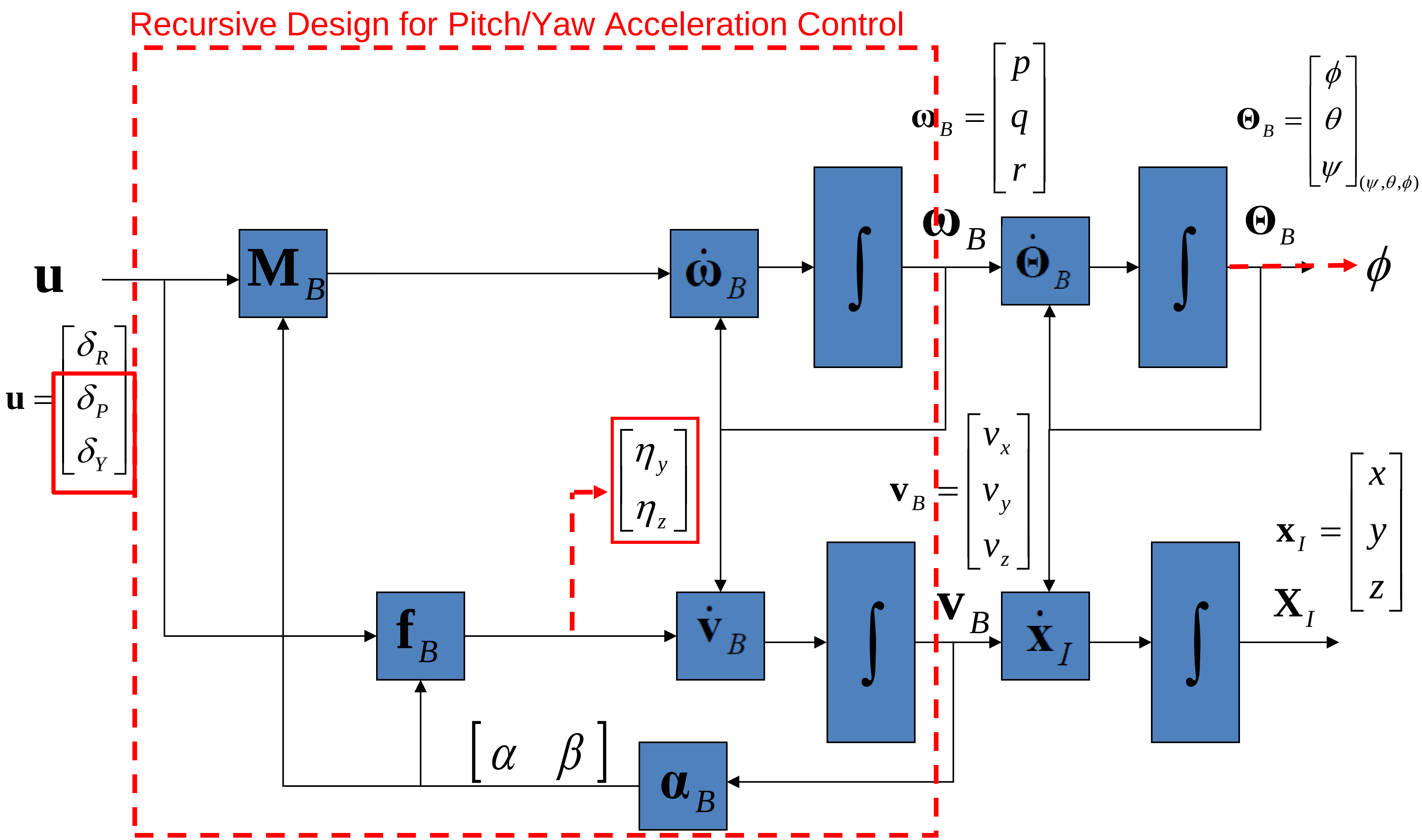
By differentiating u (n-r) times to create additional state variables, such that:

$$z_i = z_i\left(x, u, \dot{u}, \dots, u^{(n-r-1)}\right)$$

Until the control design that "pops" out is a derivative of u:

$$z_1 = x_1 - x_1^d$$
$$z_2 = \dot{z}_1 - \dot{z}_1^d$$
$$z_3 = \dot{z}_2 - \dot{z}_2^d$$
$$\vdots$$
$$z_n = \dot{z}_{n-1} - \dot{z}_{n-1}^d$$
$$0 = \dot{z}_n - \dot{z}_n^d \Rightarrow u^{(n-r)} = \dots$$

6-DOF RIGID BODY EQUATIONS OF MOTION



6-DOF RIGID BODY EQUATIONS OF MOTION

12 state equations in body coordinates:

$$\dot{\mathbf{v}}_B = \frac{1}{m}\left(\mathbf{f}_B - \dot{m}\mathbf{v}_B\right) - \boldsymbol{\omega}_B \otimes \mathbf{v}_B$$

$\mathbf{f}_B =$ External body forces

$m =$ mass

$$\dot{\boldsymbol{\omega}}_B = \mathbf{J}_B^{-1}\left(\mathbf{M}_B - \dot{\mathbf{J}}_B\boldsymbol{\omega}_B - \boldsymbol{\omega}_B \otimes \mathbf{J}_B\boldsymbol{\omega}_B\right)$$

Using the Euler Rate equations:

$\mathbf{M}_B =$ External body moments

$\mathbf{J}_B =$ Moment of Inertia Matrix

$$\dot{\boldsymbol{\Theta}}_B = \begin{bmatrix} \dot{\phi} \\ \dot{\theta} \\ \dot{\psi} \end{bmatrix}_{(\psi,\theta,\phi)} = \begin{bmatrix} p + \tan\theta(q\sin\phi + r\cos\phi) \\ q\cos\phi - r\sin\phi \\ \frac{1}{\cos\theta}(q\sin\phi + r\cos\phi) \end{bmatrix}$$

$$\dot{\mathbf{x}}_I = \mathbf{T}_{BI}^{-1}\mathbf{v}_B$$

6-DOF RIGID BODY EQUATIONS OF MOTION

The external body forces are computed as functions of:

$$\mathbf{f}_B = \begin{bmatrix} F_x(\alpha,\beta,M,\bar{q},\delta_p,\delta_y,\delta_r) \\ F_y(\alpha,\beta,M,\bar{q},\delta_p,\delta_y) \\ F_z(\alpha,\beta,M,\bar{q},\delta_p,\delta_y) \end{bmatrix} \approx \begin{bmatrix} F_{x_{BT}}(\alpha,\beta,M,\bar{q}) \\ F_{y_{BT}}(\alpha,\beta,M,\bar{q}) \\ F_{z_{BT}}(\alpha,\beta,M,\bar{q}) \end{bmatrix} + \begin{bmatrix} F_{x_i}(\alpha,\beta,M,\bar{q},\delta_r) \\ F_{y_i}(\alpha,\beta,M,\bar{q},\delta_y) \\ F_{z_i}(\alpha,\beta,M,\bar{q},\delta_p) \end{bmatrix}$$

Body Tail + Canard Control Increments

where:

α = Angle of Attack

β = Angle of Side Slip

M = Mach Number,

$\bar{q}$ = dynmaic pressure

$$\begin{bmatrix} \alpha \\ \beta \end{bmatrix} = \begin{bmatrix} \tan^{-1}\left(\dfrac{v_x}{v_z}\right) & \sin^{-1}\left(\dfrac{v_y}{v_T}\right) \end{bmatrix}^{\mathrm{T}} \qquad v_T = \sqrt{v_x^2 + v_y^2 + v_z^2}$$

$$\mathbf{T}_{BI}^{-1} = \mathbf{T}_{BI}^{T} = \begin{bmatrix} \cos\theta\cos\psi & -\cos\phi\sin\psi + \sin\phi\sin\theta\cos\psi & \sin\phi\sin\psi + \cos\phi\sin\theta\cos\psi \\ \cos\theta\sin\psi & \cos\phi\cos\psi + \sin\phi\sin\theta\sin\psi & -\sin\phi\cos\psi + \cos\phi\sin\theta\sin\psi \\ -\sin\theta & \sin\phi\cos\theta & \cos\phi\cos\theta \end{bmatrix}$$

Recursive Design for Coupled Pitch/Yaw Controller

Reference: **Dynamic Robust Recursive Control: Theory and Applications**, Ph.D. Dissertation, Richard A. Hull, University of Central Florida, Orlando Florida, 1996.

First, Define Output Error States:

$$z_1 = \begin{bmatrix} a_z - a_{zD} \\ a_y - a_{yD} \end{bmatrix}$$

$a_{zD} \equiv$ desired pitch acceleration
$a_{yD} \equiv$ desired yaw acceleration
$a_z =$ pitch acceleration feedback
$a_y =$ yaw acceleration feedback

Then, recursively define the state transformations:

$$z_1 = x_1 - x_1^d$$

$$z_2 = \dot{z}_1 - \dot{z}_1^d + \nu_1$$

$$0 = \dot{z}_2 - \dot{z}_2^d + \nu_2 \Rightarrow u = \ldots$$

Where ν_1, ν_2 **represent robust fictitious control terms to overcome bounded uncertainties.**

Recursive Design for Coupled Pitch/Yaw Controller

Choosing the desired dynamics to map to a stable linear system:

$$\dot{z}_1^d = -k_1 z_1 \quad , \quad \dot{z}_2^d = -k_2 z_2$$

Design gains: k_1, k_2

The Design Equation (without robust terms) is:

$$\ddot{z}_1 + (k_1 + k_2)\dot{z}_1 + k_1 k_2 z_1 = 0$$

Where: $\ddot{z}_1 = \begin{bmatrix} \ddot{\mathrm{a}}_z \\ \ddot{\mathrm{a}}_y \end{bmatrix}, \quad \dot{z}_1 = \begin{bmatrix} \dot{\mathrm{a}}_z \\ \dot{\mathrm{a}}_y \end{bmatrix}$

Assume that $\alpha, \beta, \bar{q}, m, v_m$ **are known parameters,**

and define the body normal accelerations to be:

$$\mathbf{a}_B = \begin{bmatrix} \mathrm{a}_z \\ \mathrm{a}_y \end{bmatrix} = \frac{1}{m} \begin{bmatrix} \mathrm{F}_z \\ \mathrm{F}_y \end{bmatrix}$$

Where Fz and Fy are the aerodynamic forces normal to the body x axis.

We write the Recursive Design Equation for an acceleration controller:

$$\ddot{\mathbf{a}}_B + (k_1 + k_2)\dot{\mathbf{a}}_B + k_1 k_2 \left(\mathbf{a}_B - \mathbf{a}^d{}_B\right) = 0$$

Recursive Design for Coupled Pitch/Yaw Controller

Now, assume that Mach and Dynamic Pressure are relatively slowing changing parameters and that the normal force contribution from delta dots is negligible, the time differential of the acceleration equations can be represented by:

$$\begin{bmatrix} \dot{a}_z \\ \dot{a}_y \end{bmatrix} \approx \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix}$$

Slopes from Aero Functions

Assume to be zero or negligible

Differentiate again to get:

$$\begin{bmatrix} \ddot{a}_z \\ \ddot{a}_y \end{bmatrix} = \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} + \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial^2 F_{z_{BT}}}{\partial \alpha^2} & \frac{2\partial^2 F_{z_{BT}}}{\partial \alpha \partial \beta} & \frac{\partial^2 F_{z_{BT}}}{\partial \beta^2} \\ \frac{\partial^2 F_{y_{BT}}}{\partial \alpha^2} & \frac{2\partial^2 F_{y_{BT}}}{\partial \alpha \partial \beta} & \frac{\partial^2 F_{y_{BT}}}{\partial \beta^2} \end{bmatrix} \begin{bmatrix} \dot{\alpha}^2 \\ \dot{\alpha}\dot{\beta} \\ \dot{\beta}^2 \end{bmatrix} + \ldots$$

Recursive Design for Coupled Pitch/Yaw Controller

Using just the pitch and yaw dynamic equations and assuming p = 0, we can compute the rate of change in alpha and beta:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \begin{bmatrix} 1 & -\sin\alpha\tan\beta \\ 0 & -\cos\alpha \end{bmatrix} \begin{bmatrix} q \\ r \end{bmatrix} + \begin{bmatrix} 0 & \left(\dfrac{\cos\alpha}{v_m\cos\beta}\right) \\ \left(\dfrac{\cos\beta}{v_m}\right) & \left(\dfrac{-\sin\alpha\sin\beta}{v_m}\right) \end{bmatrix} \begin{bmatrix} a_y \\ a_z \end{bmatrix}$$

Then, assuming small angle approximations for alpha and beta:

$$\begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \underbrace{\begin{bmatrix} q \\ -r \end{bmatrix}}_{\boldsymbol{\omega}_B} + \left(\frac{1}{v_m}\right) \begin{bmatrix} a_z \\ a_y \end{bmatrix}$$

And ...

$$\begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} = \underbrace{\begin{bmatrix} \dot{q} \\ -\dot{r} \end{bmatrix}}_{\dot{\boldsymbol{\omega}}_B} + \left(\frac{1}{v_m}\right) \begin{bmatrix} \dot{a}_z \\ \dot{a}_y \end{bmatrix}$$

Recursive Design for Coupled Pitch/Yaw Controller

Using approximations for the time derivatives of alpha and beta:

$$\begin{bmatrix} \dot{a}_z \\ \dot{a}_y \end{bmatrix} \approx \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \dot{\alpha} \\ \dot{\beta} \end{bmatrix} = \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} q + \frac{a_z}{v_m} \\ -r + \frac{a_y}{v_m} \end{bmatrix}$$

And …

$$\begin{bmatrix} \ddot{a}_z \\ \ddot{a}_y \end{bmatrix} = \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \ddot{\alpha} \\ \ddot{\beta} \end{bmatrix} = \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \dot{q} \\ -\dot{r} \end{bmatrix} + \left(\frac{1}{m v_m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \dot{a}_z \\ \dot{a}_y \end{bmatrix}$$

$$= \left(\frac{1}{m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix} \begin{bmatrix} \dot{q} \\ -\dot{r} \end{bmatrix} + \left(\frac{1}{m^2 v_m}\right) \begin{bmatrix} \frac{\partial F_z}{\partial \alpha} & \frac{\partial F_z}{\partial \beta} \\ \frac{\partial F_y}{\partial \alpha} & \frac{\partial F_y}{\partial \beta} \end{bmatrix}^2 \begin{bmatrix} q + \frac{a_z}{v_m} \\ -r + \frac{a_y}{v_m} \end{bmatrix}$$

We will solve for the control input to give the required body angular accelerations …

Recursive Design for Coupled Pitch/Yaw Controller

Recalling the recursive design equation,

$$\ddot{\mathbf{a}}_B + (k_1 + k_2)\dot{\mathbf{a}}_B + k_1 k_2 \left(\mathbf{a}_B - \mathbf{a}^d{}_B\right) = 0$$

and using the pitch-yaw acceleration and body rate vectors:

$$\mathbf{a}_B = \begin{bmatrix} \mathrm{a}_Z \\ \mathrm{a}_Y \end{bmatrix}, \qquad \boldsymbol{\omega}_B = \begin{bmatrix} q \\ -r \end{bmatrix}$$

We can substitute the preceding results into the design equation

$$\left(\frac{1}{m}\right)\mathbf{F}\dot{\boldsymbol{\omega}}_B + \left(\frac{1}{m^2 v_m}\right)\mathbf{F}^2\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) + (k_1 + k_2)\left(\frac{1}{m}\right)\mathbf{F}\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) + k_1 k_2\left(\mathbf{a}_B - \mathbf{a}^d{}_B\right) = 0$$

Where we have defined

$$\mathbf{F} = \begin{bmatrix} \dfrac{\partial F_z}{\partial \alpha} & \dfrac{\partial F_z}{\partial \beta} \\ \dfrac{\partial F_y}{\partial \alpha} & \dfrac{\partial F_y}{\partial \beta} \end{bmatrix}$$

Recursive Design for Coupled Pitch/Yaw Controller

Now, provided F is invertible, we can solve the recursive design equation, for the required body angular accelerations:

$$\dot{\boldsymbol{\omega}}^R{}_B = -\left(\frac{1}{m v_m}\right)\mathbf{F}\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) - (k_1 + k_2)\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) - k_1 k_2 m \mathbf{F}^{-1}\left(\mathbf{a}_B - \mathbf{a}^d{}_B\right)$$

Equating to the body moment equations:

$$\dot{\boldsymbol{\omega}}_B = \mathbf{J}_B^{-1}(\mathbf{M}_B - \boldsymbol{\omega}_B \otimes \mathbf{J}_B \boldsymbol{\omega}_B)$$

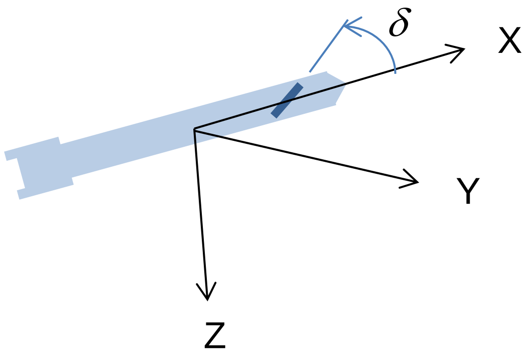
We can solve for the body moments required to give the desired body acceleration response:

$$\mathbf{M}^R{}_B = \mathbf{J}_B \dot{\boldsymbol{\omega}}^R{}_B + \boldsymbol{\omega}_B \otimes \mathbf{J}_B \boldsymbol{\omega}_B$$

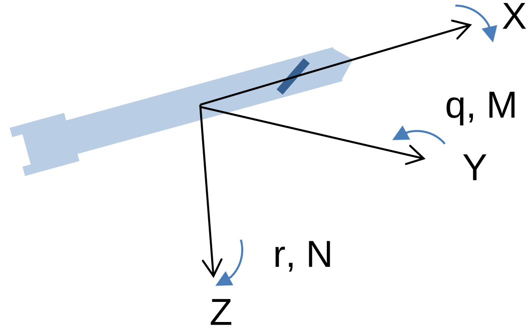
Then, we “invert” the aerodynamic moment equations to find the pitch and yaw control input deflections that will give the required body moments above.

Conventions for Aerodynamic Definitions

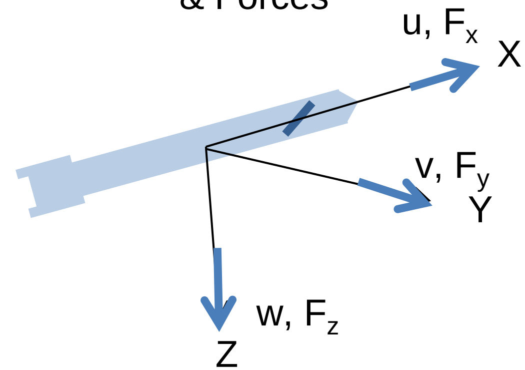
Body Axes



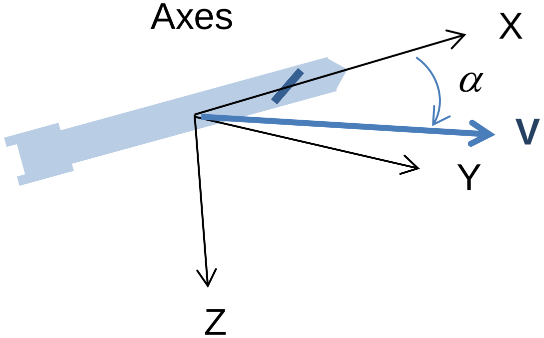
Body Rates
& Moments



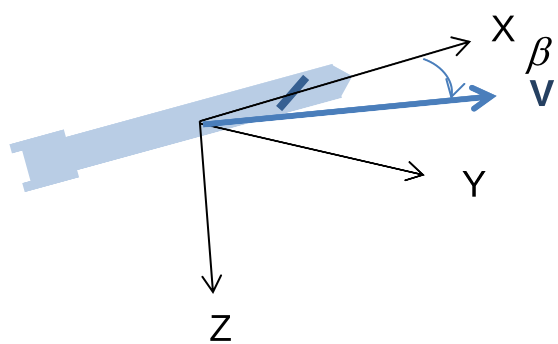
Body Velocities,
& Forces



Pitch Plane
Stability
Axes



Yaw Plane
Stability Axes



where :

α = Angle of Attack

β = Angle of Side Slip

δ = Canard Deflection,

p, q, r = Body Rates

u, v, w = Body Velocities

L, M, N = Body Moments

F_x, F_y, F_z = Body Forces

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Hypothetical Projectile Aerodynamics Model

Define aerodynamic forces and moments in the pitch and yaw axes in terms of alpha, beta, and the pitch and yaw control deflections as:

$$F_z = \bar{q} S_{ref} C_Z(\alpha, \beta, \delta_P) \qquad \text{Aero Force along body z axis}$$

$$F_y = \bar{q} S_{ref} C_Y(\alpha, \beta, \delta_Y) \qquad \text{Aero Force along body y axis}$$

$$m = \bar{q} S_{ref} l_{ref} C_m(\alpha, \beta, \delta_P) \qquad \text{Aero Pitching Moment}$$

$$n = \bar{q} S_{ref} l_{ref} C_n(\alpha, \beta, \delta_Y) \qquad \text{Aero Yawing Moment}$$

Where: $\bar{q} = \text{dynamic pressure},\ S_{ref} = \text{reference area},\ l_{ref} = \text{reference length}$

Assume aerodynamic coefficients are the following functions of alpha, beta, pitch and yaw control deflections (in degrees):

$$C_Z(\alpha, \beta, \delta_P) = .000103\alpha^3 - .00945\alpha|\alpha| - .017\alpha - .00055\alpha^2\beta - .034\delta_P$$

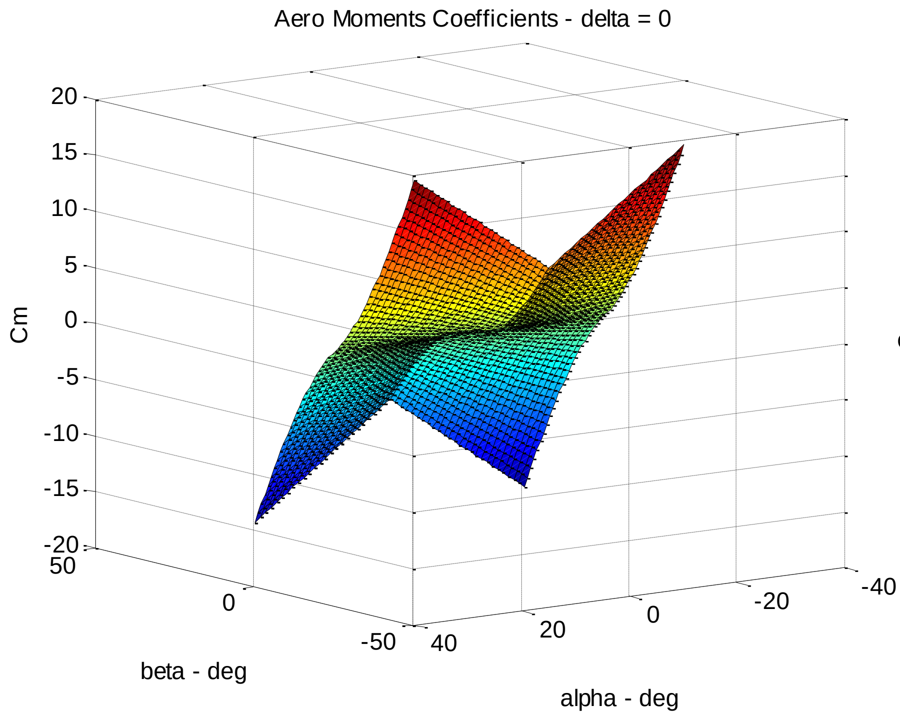
$$C_Y(\alpha, \beta, \delta_Y) = .000103\beta^3 - .00945\beta|\beta| - .017\beta - .00055\beta^2\alpha - .034\delta_Y$$

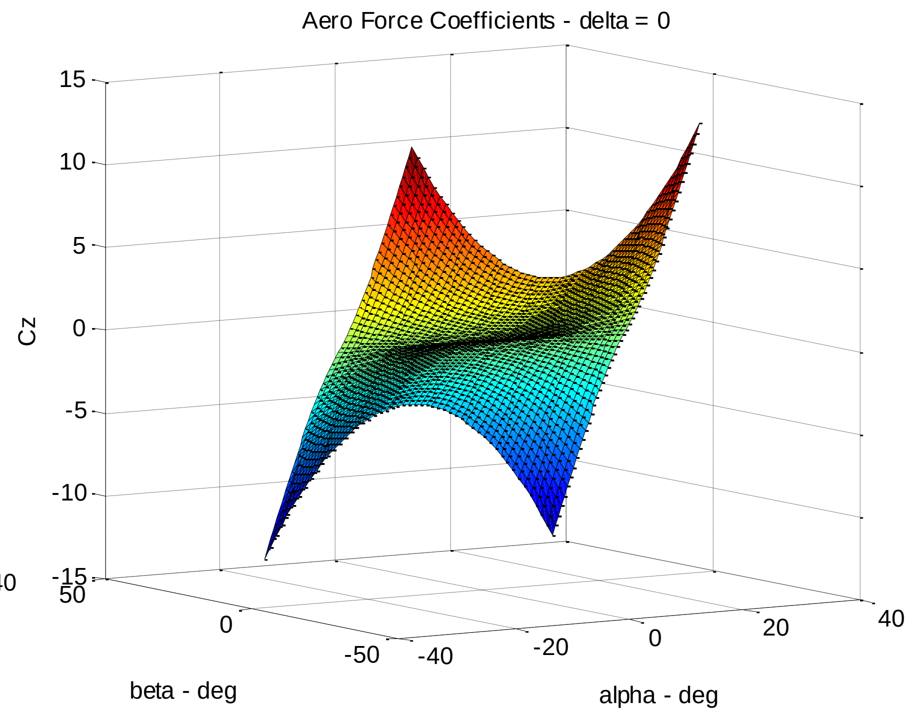
$$C_m(\alpha, \beta, \delta_P) = .000215\alpha^3 - .0195\alpha|\alpha| + .051\alpha - .015\alpha|\beta| + .700\delta_P$$

$$C_n(\alpha, \beta, \delta_Y) = .000215\beta^3 - .0195\beta|\beta| + .051\beta - .015\beta|\alpha| + .700\delta_Y$$

Coupled Nonlinear Equations in Alpha and Beta

Aerodynamic Force and Moment Coefficients for a Hypothetical Projectile as Function of Alpha, Beta for Delta = 0





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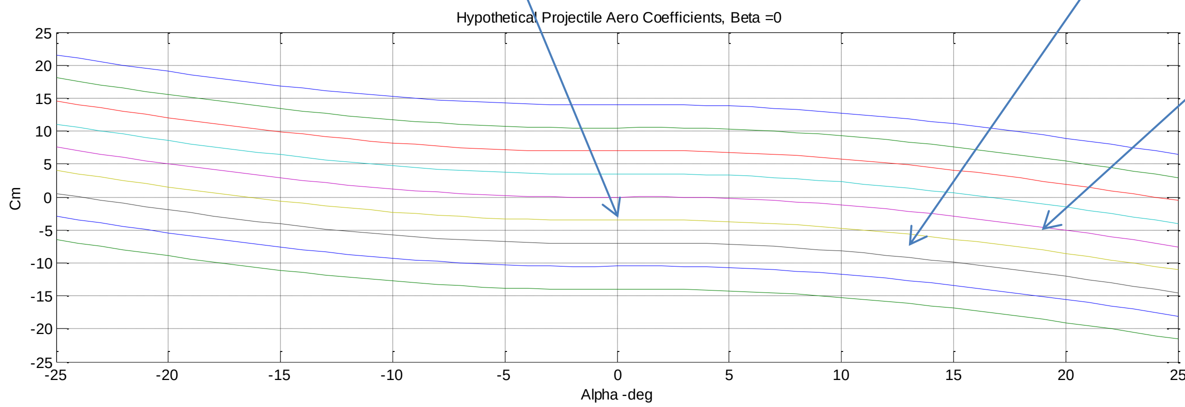
Aerodynamic Force and Moment Coefficients for a Hypothetical Projectile as Function of Alpha, Beta and Delta

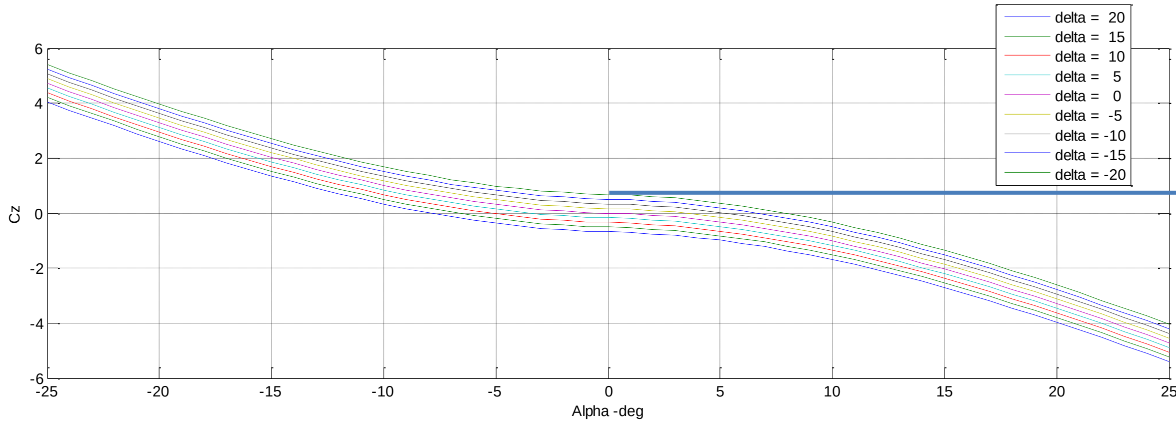
Neutrally stable with zero delta around alpha = 0

-Cm at +alpha indicates stable airframe

10 deg delta to trim at 15 deg alpha



+ delta = + moment



+ Alpha = - Fz

-20 deg

+20 deg

+ delta = - accel

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Recursive Pitch/Yaw Controller for the Hypothetical Projectile

Using the Force Coefficient Polynomials:

$$C_Z(\alpha,\beta,\delta_P) = a_1\alpha^3 + a_2\alpha|\alpha| + a_3\alpha + a_4\alpha^2\beta + b_1\delta_P$$

$$C_Y(\alpha,\beta,\delta_Y) = a_1\beta^3 + a_2\beta|\beta| + a_3\beta + a_4\beta^2\alpha + b_1\delta_Y$$

$$\text{where: } a_1 = 0.000103,\quad a_2 = -0.00945,\quad a_3 = -0.017,\quad a_4 = -0.00055,\quad b_1 = -0.034$$

We can find the required "aero slopes" matrix:

$$\mathbf{F} = \begin{bmatrix} \dfrac{\partial F_z}{\partial \alpha} & \dfrac{\partial F_z}{\partial \beta} \\ \dfrac{\partial F_y}{\partial \alpha} & \dfrac{\partial F_y}{\partial \beta} \end{bmatrix} = \bar{q} S_{ref} \begin{bmatrix} \dfrac{\partial C_z}{\partial \alpha} & \dfrac{\partial C_z}{\partial \beta} \\ \dfrac{\partial C_y}{\partial \alpha} & \dfrac{\partial C_y}{\partial \beta} \end{bmatrix} \left(\frac{180}{\pi}\right)$$

because (α, β) are in degrees!

From the coefficient polynomials:

$$\frac{\partial C_Z}{\partial \alpha} = 3a_1\alpha^2 + 2a_2\alpha\,\mathrm{sign}(\alpha) + a_3 + 2a_4\alpha\beta \quad , \quad \frac{\partial C_Z}{\partial \beta} = a_4\alpha^2$$

$$\frac{\partial C_Y}{\partial \alpha} = a_4\beta^2 \quad , \quad \frac{\partial C_Y}{\partial \beta} = 3a_1\beta^2 + 2a_2\beta\,\mathrm{sign}(\beta) + a_3 + 2a_4\beta\alpha$$

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Recursive Pitch/Yaw Controller for the Hypothetical Projectile

Using F, we compute the required body angular rates and moments:

$$\dot{\boldsymbol{\omega}}^R{}_B = -\left(\frac{1}{mv_m}\right)\mathbf{F}\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) - (k_1 + k_2)\left(\boldsymbol{\omega}_B + \frac{\mathbf{a}_B}{v_m}\right) - k_1 k_2 m \mathbf{F}^{-1}\left(\mathbf{a}_B - \mathbf{a}^d{}_B\right)$$

$$\mathbf{M}^R{}_B = \mathbf{J}_B \dot{\boldsymbol{\omega}}^R{}_B + \cancel{\boldsymbol{\omega}_B \otimes \mathbf{J}_B \boldsymbol{\omega}_B}$$

Ignore – pitch and yaw components are zero if p = 0

where: $\mathbf{a}_B = \begin{bmatrix} \mathrm{a}_Z \\ \mathrm{a}_Y \end{bmatrix}$, $\boldsymbol{\omega}_B = \begin{bmatrix} q \\ -r \end{bmatrix}$ are from the accelerometer and gyro feedbacks

the following parameters are estimated in real-time:

α, β = Angle of Attack and Angle of Side Slip

$\bar{q}$ = dynmaic pressure

v_m = velocity of projectile

m = mass of projectile

$\mathbf{J}_B$ = pitch/yaw moments of intetial $= \begin{bmatrix} \mathrm{J}_{zz} & \mathrm{J}_{yz} \\ \mathrm{J}_{yz} & \mathrm{J}_{yy} \end{bmatrix}$

Inertia Coupling Included

Note – in the autopilot we have let w_b(2) = -r , therefore reverse the sign of the required yaw moment!

And k_1, k_2 are gains chosen by the designer to place the poles of the feedback linearized system.

Recursive Design Applied to the Hypothetical Projectile

Finally, we use the aerodynamic moment equations:

$$C_m(\alpha,\beta,\delta_P) = c_1\alpha^3 + c_2\alpha|\alpha| + c_3\alpha + c_4\alpha|\beta| + d_1\delta_P$$
$$C_n(\alpha,\beta,\delta_Y) = c_1\beta^3 + c_2\beta|\beta| + c_3\beta + c_4\beta|\alpha| + d_1\delta_Y$$

Note: Pitch and Yaw Control Contributions May Not be De-coupled in the Real World!

$$\text{where: } c_1 = 0.000215, \quad c_2 = -0.0195, \quad c_3 = 0.051, \quad c_4 = -0.015, \quad d_1 = 0.700$$

to solve for the pitch and yaw control deflections needed to give the required pitch and yaw moments:

$$\mathbf{M}_B^R = \begin{bmatrix} M_p^R \\ M_Y^R \end{bmatrix} = \bar{q}S_{ref}l_{ref}\begin{bmatrix} C_m(\alpha,\beta,\delta_P) \\ C_n(\alpha,\beta,\delta_Y) \end{bmatrix}$$

so the pitch and yaw control deflection commands are:

$$\delta_P = \frac{1}{d_1}\left[\frac{1}{\bar{q}S_{ref}l_{ref}}M_p^R - \left(c_1\alpha^3 + c_2\alpha|\alpha| + c_3\alpha + c_4\alpha|\beta|\right)\right]$$

$$\delta_Y = \frac{1}{d_1}\left[-\frac{1}{\bar{q}S_{ref}l_{ref}}M_Y^R - \left(c_1\beta^3 + c_2\beta|\beta| + c_3\beta + c_4\beta|\alpha|\right)\right]$$

Note – in the autopilot we have let w_b(2) = -r , therefore reverse the sign of the required yaw moment!

where again the parameters: $\alpha, \beta, \bar{q}$, are estimated in real-time, and S_{ref} and l_{ref} are the known reference area and length.

Hypothetical Projectile Parameters

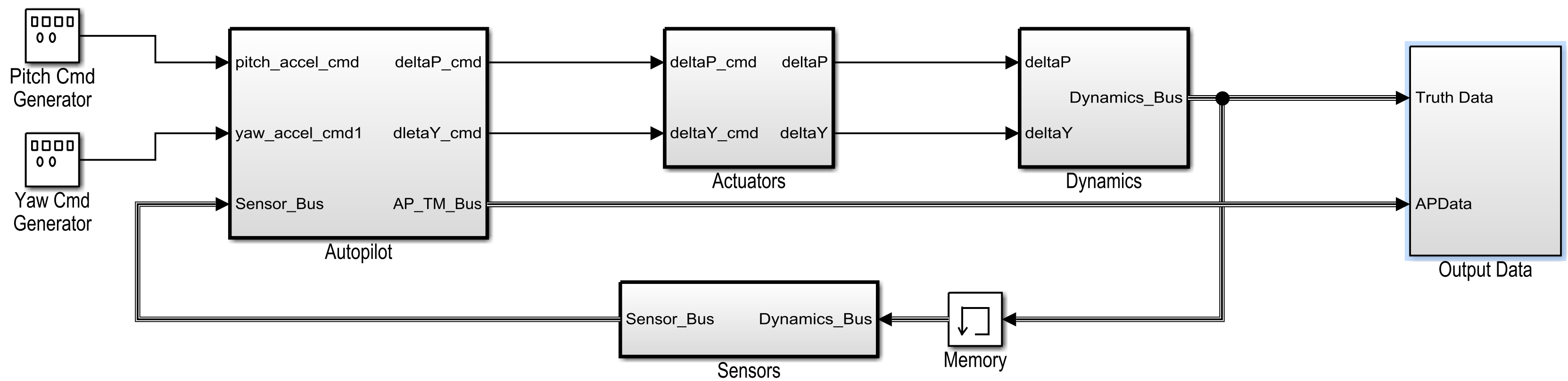
A totally fictitious configuration intended as a model to study guidance and control issues for guided projectiles.

Model Parameters

Parameter	Value	Units
Diameter	140	mm
Mass	50	kg
Xcg (from nose)	1.0	m
Ixx	0.200	Kg-m^2
Iyy = Izz	18.00	Kg-m^2
Iyz = Izy	0.50	Kg-m^2
Sref (aero reference area)	0.0154	m^2
Lref (aero reference length)	1.0	m
MRC (aero model ref center)	1.0	m

MATLAB/Simulink Simulation

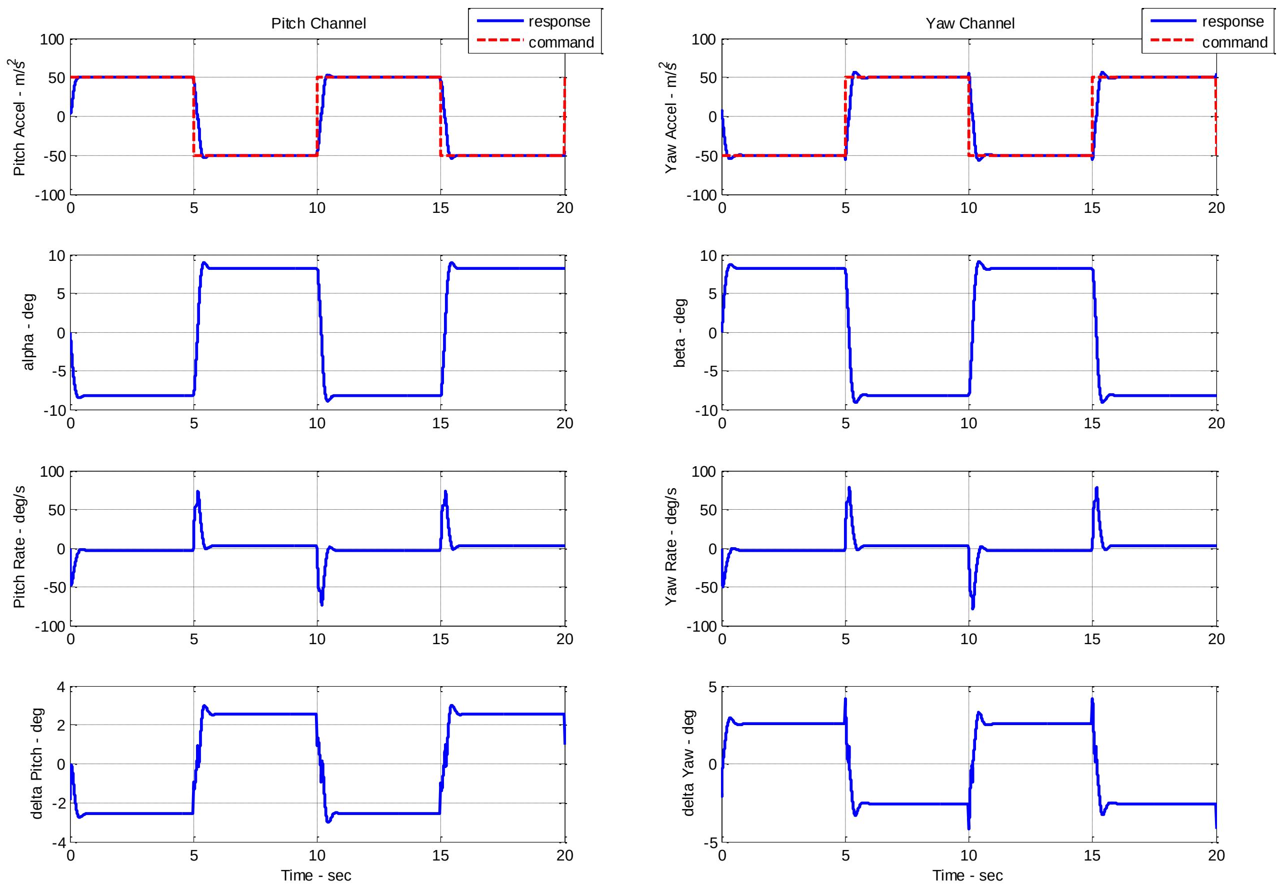
Hypothetical Projectile
Simplified Pitch/Yaw Controller Simulation
- Rich Hull -



Unclassified
This Document Does Not Contain Any Export Controlled Technical Data

MATLAB/Simulink Simulation Results

Simultaneous Pitch and Yaw Step Commands
Full Coupled Nonlinear Aero Model with Inertia Coupling



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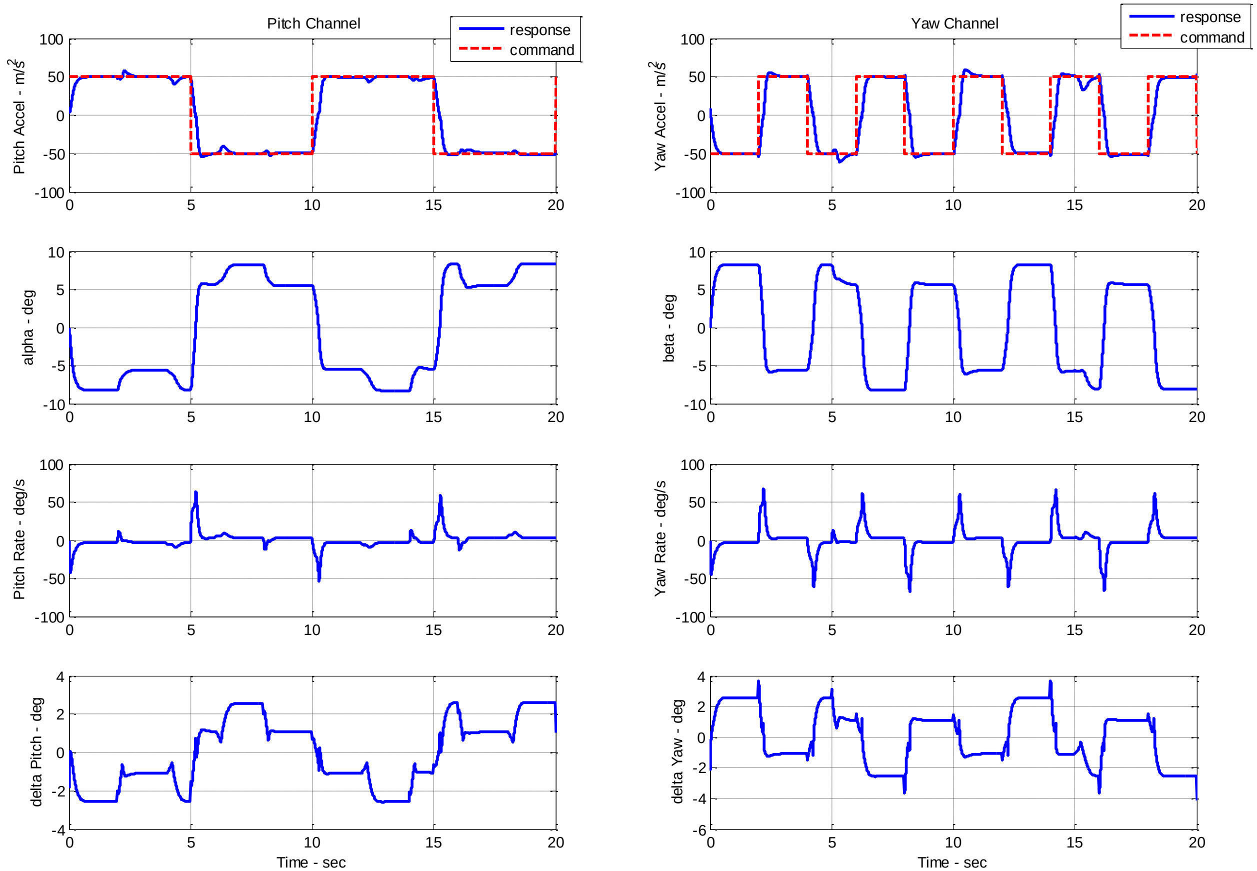
MATLAB/Simulink Simulation Results

Simultaneous Pitch and Yaw Step Commands at Different Frequencies
Full Coupled Nonlinear Aero Model with Inertia Coupling



Summary and Conclusions

- Precision Guided Projectiles Pose Challenging Guidance, Navigation and Control Problems
- GENEX Guidance Law Provides an Analytical Solution to the Optimal Guidance Problem
- Nonlinear Recursive Control Design Approach Demonstrated to Get Analytical Solution for Pitch/Yaw Autopilot for a Hypothetical Projectile with Coupled Nonlinear Aerodynamics and Off-Diagonal Inertia Coupling Terms.
  - Method requires computation of “slopes” of aerodynamic force functions, and inverse of aero moment functions
  - Good tracking control can be achieved without addition of integrators to controller
  - Good method for getting quick “simulation quality” controller
  - Problems may occur if aerodynamic partial derivatives matrix becomes ill-conditioned
  - Additional terms can be added to provide robustness to bounded uncertainties and un-modelled nonlinear dynamics

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