Robust Adaptive Control of Multi-Rotor UAVs

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Unmanned Aerial Vehicles – UAVs

Typical UAV tasks include:

- Surveillance;
- Crop monitoring;
- Search and rescue;

Mission scenarios usually

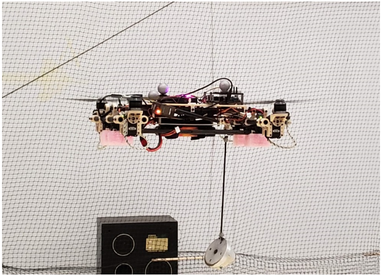
- Occur in open space;
- Require calm weather conditions;
- Assume known vehicle and payload properties

These are passive missions in the environment

UAVs of the Future

Intelligent UAVs of the future will actively interact with the environment performing tasks such as

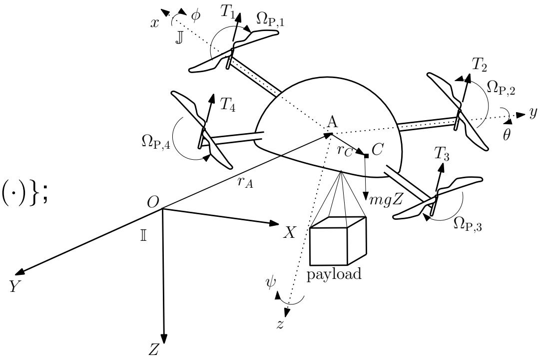
- Maneuvering unknown objects;
- Interacting with walls or hard surfaces;
    - Aerodynamic wall effects;
- Pulling or pushing items such as cables



Novel theoretical frameworks needed to design suitable autopilots

Modeling Assumptions

- Center of mass is not the reference point $A(\cdot)$;
- Main frame is rigid;
- Inertia matrix is constant
- Inertial reference frame,
  - $\mathbb{I} = \{O; X, Y, Z\}$;
- Body reference frame,
  - $\mathbb{J}(\cdot) = \{A(\cdot); x(\cdot), y(\cdot), z(\cdot)\}$;
- Position of $A(\cdot)$ w.r.t O,
  - $r_A^{\mathbb{I}} : [t_0, \infty) \to \mathbb{R}^3$;
- Velocity of $A(\cdot)$ w.r.t $\mathbb{I}$,
  - $v_A^{\mathbb{I}} : [t_0, \infty) \to \mathbb{R}^3$;
- Angular velocity of $\mathbb{J}(\cdot)$ w.r.t $\mathbb{I}$,
  - $\omega : [t_0, \infty) \to \mathbb{R}^3$;



Kinematic Equations

Translational Kinematic Equations

$$\dot{r}_A^{\mathbb{I}}(t) = R(\phi(t), \theta(t), \psi(t)) v_A(t), \qquad r_A^{\mathbb{I}}(t_0) = r_{A,0}^{\mathbb{I}}, \qquad t \geq t_0,$$

$$R(\phi, \theta, \psi) \triangleq \begin{bmatrix} \cos\psi & -\sin\psi & 0 \\ \sin\psi & \cos\psi & 0 \\ 0 & 0 & 1 \end{bmatrix} \begin{bmatrix} \cos\theta & 0 & \sin\theta \\ 0 & 1 & 0 \\ -\sin\theta & 0 & \cos\theta \end{bmatrix} \begin{bmatrix} 1 & 0 & 0 \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi & \cos\phi \end{bmatrix},$$

$$(\phi, \theta, \psi) \in [0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right) \times [0, 2\pi)$$

Rotational Kinematic Equations

$$\begin{bmatrix} \dot{\phi}(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \end{bmatrix} = \Gamma(\phi(t), \theta(t)) \omega(t), \qquad \begin{bmatrix} \phi(t_0) \\ \theta(t_0) \\ \psi(t_0) \end{bmatrix} = \begin{bmatrix} \phi_0 \\ \theta_0 \\ \psi_0 \end{bmatrix},$$

$$\Gamma(\phi, \theta) \triangleq \begin{bmatrix} 1 & \sin\phi\tan\theta & \cos\phi\tan\theta \\ 0 & \cos\phi & -\sin\phi \\ 0 & \sin\phi\sec\theta & \cos\phi\sec\theta \end{bmatrix}$$

Dynamic Equations I

- Translational Dynamic Equations

$$
\begin{aligned}
&F_{\mathrm{g}}(\phi(t),\theta(t)) - F_{\mathrm{T}}(t) + F(t) \\
&\qquad = m\left[\dot{v}_A(t) + \omega^{\times}(t)v_A(t)\right] + m\left[\ddot{r}_C(t) + \dot{\omega}^{\times}(t)r_C(t)\right. \\
&\qquad\qquad \left. + 2\omega^{\times}(t)\dot{r}_C(t) + \omega^{\times}(t)\omega^{\times}(t)r_C(t)\right], \qquad v_A(t_0) = v_{A,0},
\end{aligned}
$$

  - Thrust force: $-F_{\mathrm{T}} = [0, 0, u_1]^{\mathrm{T}}$,
  - Quadcopter's weight

$$
F_{\mathrm{g}}(\phi,\theta) = mg[-\sin\theta, \cos\theta\sin\phi, \cos\theta\cos\phi]^{\mathrm{T}},
$$
$$
(\phi,\theta) \in [0, 2\pi) \times \left(-\frac{\pi}{2}, \frac{\pi}{2}\right),
$$

  - Aerodynamic drag: $F : [t_0, \infty) \to \mathbb{R}^3$

Dynamic Equations II

Rotational Dynamic Equations

$$
\begin{aligned}
M_{\mathrm{T}}(t) + M_{\mathrm{g}}(r_C(t), \phi(t), \theta(t)) + M(t) \\
= m r_C^{\times}(t)\left[\dot{v}_A(t) + \omega^{\times}(t) v_A(t)\right] + I\dot{\omega}(t) + \omega^{\times}(t) I \omega(t) \\
+ I_{\mathrm{P}} \sum_{i=1}^{4} \begin{bmatrix} 0 \\ 0 \\ \dot{\Omega}_{\mathrm{P},i}(t) \end{bmatrix} + \omega^{\times}(t) I_{\mathrm{P}} \sum_{i=1}^{4} \begin{bmatrix} 0 \\ 0 \\ \Omega_{\mathrm{P},i}(t) \end{bmatrix}, \\
\omega(t_0) = \omega_0, \qquad t \geq t_0
\end{aligned}
$$

- Moment of the propellers' forces: $M_T = [u_2, u_3, u_4]^{\mathrm{T}}$,
- Moment of the weight: $M_{\mathrm{g}}(r_C, \phi, \theta) \triangleq r_C^{\times} F_{\mathrm{g}}(\phi, \theta)$,
- Moment of the aerodynamic drag: $M : [t_0, \infty) \to \mathbb{R}^3$

Remark

Translational Dynamic Equations II

$$m\ddot{r}_A^{\mathbb{I}}(t) = R(\phi(t),\theta(t),\psi(t))\begin{bmatrix}0\\0\\u_1(t)\end{bmatrix} + \begin{bmatrix}0\\0\\mg\end{bmatrix} + F^{\mathbb{I}}(t) - m\ddot{r}_C^{\mathbb{I}}(t),$$

$$r_A^{\mathbb{I}}(t_0) = r_{A,0}^{\mathbb{I}}, \qquad v_A^{\mathbb{I}}(t_0) = v_{A,0}^{\mathbb{I}}, \qquad t \geq t_0$$

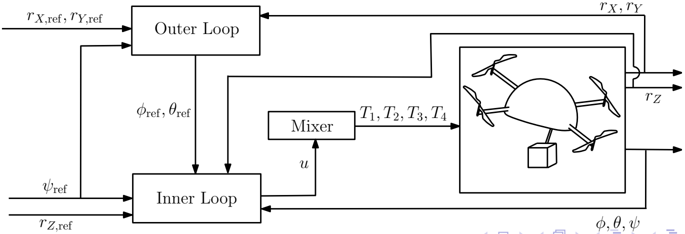
"Mixer"

$$\begin{bmatrix}u_1(t)\\u_2(t)\\u_3(t)\\u_4(t)\end{bmatrix} = \begin{bmatrix}1 & 1 & 1 & 1\\ 0 & -l & 0 & l\\ l & 0 & -l & 0\\ -c_{\mathrm{T}} & c_{\mathrm{T}} & -c_{\mathrm{T}} & c_{\mathrm{T}}\end{bmatrix}\begin{bmatrix}T_1(t)\\T_2(t)\\T_3(t)\\T_4(t)\end{bmatrix},$$

$$T_i(t) = k\Omega_{\mathrm{P},i}^2(t), \qquad i = 1,\ldots,4$$

Conventional Autopilot Architecture

- 6 states $r_A(\cdot)$, $v_A(\cdot)$, $\phi(\cdot)$, $\theta(\cdot)$, $\psi(\cdot)$
- 4 controls $u_1(\cdot), \ldots, u_4(\cdot)$
  - Quadcopters are underactuated
- Choose 4 reference values to track: $r_{X,\text{ref}}, r_{Y,\text{ref}}, r_{Z,\text{ref}}, \psi_{\text{ref}} : [t_0, \infty) \to \mathbb{R}$;
- Outer loop: Deduce $\phi_{\text{ref}}, \theta_{\text{ref}} : [t_0, \infty) \to \mathbb{R}$
- Inner loop: Deduce $u_1(\cdot), \ldots, u_4(\cdot)$



Outer Loop Design — Example 1

If $\phi(\cdot)$ & $\theta(\cdot)$ are small & $r_C(t) \equiv 0$, then

$$\begin{bmatrix} \ddot{r}_X(t) \\ \ddot{r}_Y(t) \end{bmatrix} = \frac{u_1(t)}{m} \begin{bmatrix} \sin\psi(t) & \cos\psi(t) \\ -\cos\psi(t) & \sin\psi(t) \end{bmatrix} \begin{bmatrix} \phi(t) \\ \theta(t) \end{bmatrix},$$

PID Controller

$$\begin{bmatrix} \phi_{\text{ref}}(t) \\ \theta_{\text{ref}}(t) \end{bmatrix} = \frac{m}{u_1(t)} \begin{bmatrix} \sin\psi_{\text{ref}}(t) & -\cos\psi_{\text{ref}}(t) \\ \cos\psi_{\text{ref}}(t) & \sin\psi_{\text{ref}}(t) \end{bmatrix}$$
$$\cdot \left(K_\text{p} \begin{bmatrix} r_{X,\text{ref}}(t) - r_X(t) \\ r_{Y,\text{ref}}(t) - r_Y(t) \end{bmatrix} + K_\text{d} \begin{bmatrix} \dot{r}_{X,\text{ref}}(t) - \dot{r}_X(t) \\ \dot{r}_{Y,\text{ref}}(t) - \dot{r}_Y(t) \end{bmatrix} + \begin{bmatrix} \ddot{r}_{X,\text{ref}}(t) \\ \ddot{r}_{Y,\text{ref}}(t) \end{bmatrix} \right),$$

Outer Loop Design — Example 2

If $r_C(t) \equiv 0$, then

$$\begin{bmatrix} \ddot{r}_X(t) \\ \ddot{r}_Y(t) \end{bmatrix} = \frac{u_1(t)}{m} \begin{bmatrix} u_X(t) \\ u_Y(t) \end{bmatrix}.$$

- Design $u_X(\cdot)$ & $u_Y(\cdot)$ applying *any* control technique
- Find $\phi_{\text{ref}}(\cdot)$ & $\theta_{\text{ref}}(\cdot)$ s.t.

$$\begin{aligned} \sin \phi_{\text{ref}}(t) &= u_X(t) \sin \psi_{\text{ref}}(t) - u_Y(t) \cos \psi_{\text{ref}}(t), \\ \sin \theta_{\text{ref}}(t) \cos \phi_{\text{ref}}(t) &= u_X(t) \cos \psi_{\text{ref}}(t) + u_Y(t) \sin \psi_{\text{ref}}(t) \end{aligned}$$

Model Reference Adaptive Control (MRAC)

- Consider the plant & reference model

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\mathrm{T}}\Phi(t, x(t))\right], \quad x(t_0) = x_0, \quad t \geq t_0,$$
$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref}}x_{\mathrm{ref}}(t) + B_{\mathrm{ref}}r(t), \qquad x_{\mathrm{ref}}(t_0) = x_{\mathrm{ref},0}$$

$x(t) \in \mathcal{D}$, A, Θ unknown, (A, B) controllable & A_{ref} Hurwitz,

- Assume that $\exists$ $K_x \in \mathbb{R}^{n\times m}$ & $K_r \in \mathbb{R}^{m\times m}$ s.t.

$$A_{\mathrm{ref}} = A + B\Lambda K_x^{\mathrm{T}}, \qquad B_{\mathrm{ref}} = B\Lambda K_r^{\mathrm{T}},$$

- Both K_x & K_r are unknown
- If A & Θ were known, then

$$u_{\mathrm{ideal}} = K_x^{\mathrm{T}}x + K_r^{\mathrm{T}}r - \Theta^{\mathrm{T}}\Phi(t, x)$$

would guarantee asymptotic convergence of $e(t) \triangleq x(t) - x_{\mathrm{ref}}(t)$

Goals

Find $\hat{K}_x(\cdot)$, $\hat{K}_r(\cdot)$ & $\hat{\Theta}(\cdot)$ s.t.

$$u = \hat{K}_x^{\mathrm{T}}(t)x + \hat{K}_r^{\mathrm{T}}(t)r - \hat{\Theta}^{\mathrm{T}}(t)\Phi(t,x)$$

- $e(t) \to 0$ as $t \to \infty$
- $e(\cdot)$, $\Delta K_x(\cdot)$, $\Delta K_r(\cdot)$ & $\Delta\Theta(\cdot)$ uniformly bounded

$$\Delta K_x(t) \triangleq \hat{K}_x(t) - K_x,$$
$$\Delta K_r(t) \triangleq \hat{K}_r(t) - K_r,$$
$$\Delta\Theta(t) \triangleq \hat{\Theta}(t) - \Theta,$$

Idea

Control law for $u(\cdot)$ mimics behavior of $u_{\text{ideal}}(\cdot)$

Classical MRAC

- Given $Q = Q^{\mathrm{T}} > 0$, & let $P = P^{\mathrm{T}} > 0$ s.t.

$$\boxed{0 = A_{\mathrm{ref}}^{\mathrm{T}} P + P A_{\mathrm{ref}} + Q}$$

- Let

$$\begin{aligned}
\dot{\hat{K}}_x^{\mathrm{T}} &= -\Gamma_x x(t) e^{\mathrm{T}}(t) P B, \qquad \hat{K}_x(t_0) = K_{x,0}, \qquad t \geq t_0, \\
\dot{\hat{K}}^{\mathrm{T}} &= -\Gamma_r r(t) e^{\mathrm{T}}(t) P B, \qquad \hat{K}_r(t_0) = K_{r,0}, \\
\dot{\hat{\Theta}}^{\mathrm{T}} &= \Gamma_\Theta \Phi(t, x(t)) e^{\mathrm{T}}(t) P B, \qquad \hat{\Theta}(t_0) = \Theta_0
\end{aligned}$$

- Then, $\lim_{t\to\infty} e(t) = 0$ uniformly in t_0 & $e(\cdot)$, $\Delta K_x(\cdot)$, $\Delta K_r(\cdot)$ & $\Delta\Theta(\cdot)$ uniformly bounded

e-Modification MRAC

- Consider the plant

$$\dot{x}(t) = Ax(t) + B\Lambda\left[u(t) + \Theta^{\mathrm{T}}\Phi(t, x(t))\right] + \xi(t), \quad x(t_0) = x_0,$$
$$\dot{x}_{\mathrm{ref}}(t) = A_{\mathrm{ref}}x_{\mathrm{ref}}(t) + B_{\mathrm{ref}}r(t), \qquad x_{\mathrm{ref}}(t_0) = x_{\mathrm{ref},0}$$

where $\|\xi(t)\| \leq \xi_{\max}$,

- Let

$$\dot{\hat{K}}_x^{\mathrm{T}}(t) = -\Gamma_x x(t)e^{\mathrm{T}}(t)PB - \sigma_x\|e^{\mathrm{T}}(t)PB\|\hat{K}_x^{\mathrm{T}}(t), \quad \hat{K}_x(t_0) = K_{x,0},$$
$$\dot{\hat{K}}^{\mathrm{T}}(t) = -\Gamma_r r(t)e^{\mathrm{T}}(t)PB - \sigma_r\|e^{\mathrm{T}}(t)PB\|\hat{K}_r^{\mathrm{T}}(t), \quad \hat{K}_r(t_0) = K_{r,0},$$
$$\dot{\hat{\Theta}}^{\mathrm{T}}(t) = \Gamma_\Theta\Phi(t, x(t))e^{\mathrm{T}}(t)PB + \sigma_\Theta\|e^{\mathrm{T}}(t)PB\|\hat{\Theta}^{\mathrm{T}}(t), \quad \hat{\Theta}(t_0) = \Theta_0$$

- Then, $e(\cdot)$ uniformly ultimately bounded & $\Delta K_x(\cdot)$, $\Delta K_r(\cdot)$ & $\Delta\Theta(\cdot)$ uniformly bounded

Inner Loop Design — Example

- If r_C constant & unknown, inner-loop linearized equations:

$$\begin{bmatrix} \dot{z}(t) \\ \dot{\phi}(t) \\ \dot{\theta}(t) \\ \dot{\psi}(t) \\ \dot{v}_z(t) \\ \dot{\omega}(t) \end{bmatrix} = \begin{bmatrix} 0_{4\times4} & \mathbf{1}_4 \\ \begin{bmatrix} 0 & -g & 0 & 0 \\ g & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 \end{bmatrix} & 0_{4\times4} \end{bmatrix} \begin{bmatrix} z(t) \\ \phi(t) \\ \theta(t) \\ \psi(t) \\ v_z(t) \\ \omega(t) \end{bmatrix} + \begin{bmatrix} 0_{4\times4} \\ \mathbf{1}_4 \end{bmatrix} \begin{bmatrix} -m^{-1} & 0 & 0 & 0 \\ 0 & I_x^{-1} & 0 & 0 \\ 0 & 0 & I_y^{-1} & 0 \\ 0 & 0 & 0 & I_z^{-1} \end{bmatrix}$$

$$\cdot \left[u(t) + \begin{bmatrix} 0_{1\times3} \\ r_C^\times \end{bmatrix} \left(F_g(\phi(t), \theta(t)) - m \left(\dot{v}_A(t) + \omega^\times(t) v_A(t) \right) \right) \right],$$

$$[z(0), \phi(0), \theta(0), \psi(0), v_z(0), \omega(0)]^T = [z_0, \phi_0, \theta_0, \psi_0, v_{z,0}, \omega_0]^T$$

- Same form as MRAC plant

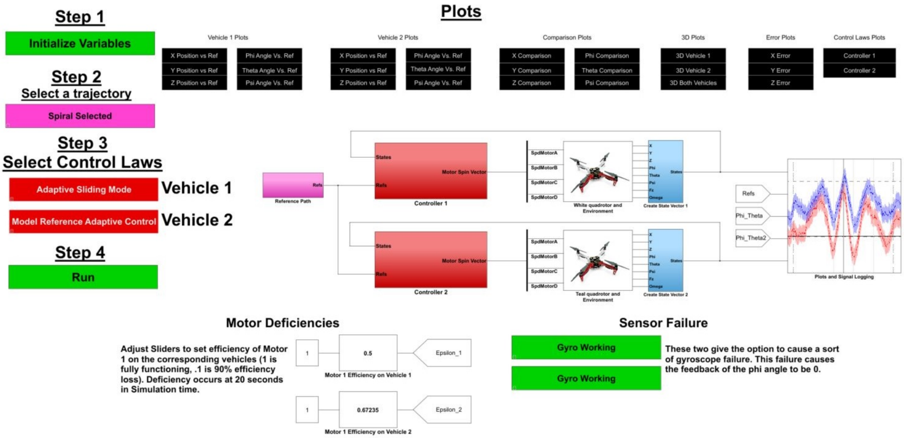
CAD-Based Simulator

Simulation Environment

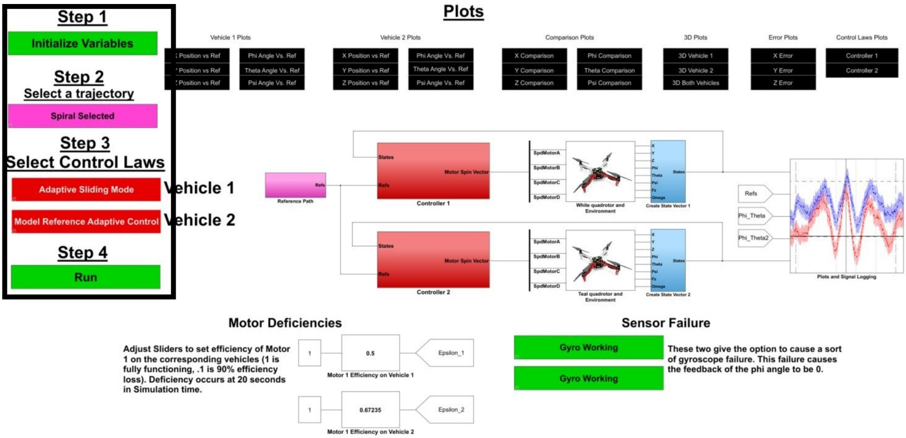
- Matlab's SimScape environment utilizes accurate CAD models;
- EoM not coded: solved from forces applied to CAD model;
- Includes sensor noise and motor models;
- Distubances: motor deficiencies and simulated gyro failures;
- Includes 4 control laws:
  - PID Control
  - Sliding Mode Control
  - Model Reference Adaptive Control
  - Adaptive Sliding Mode Control

http://lafflitto.com/Quad_Simulator.html to download simulator

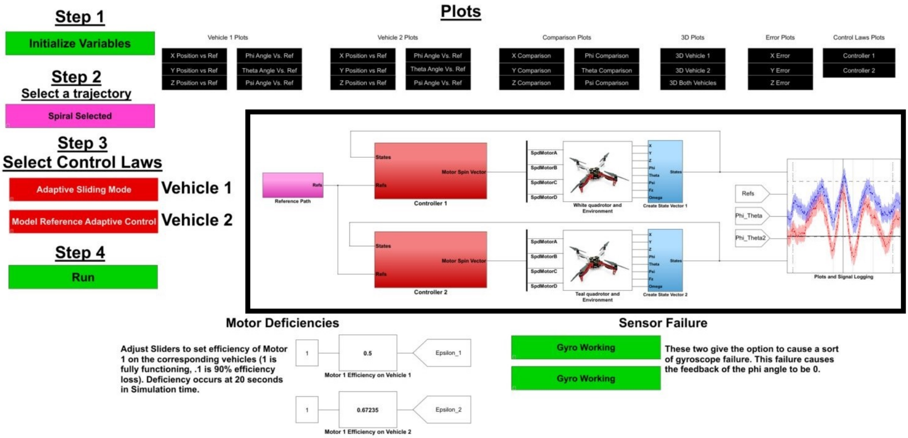
Simulator Layout



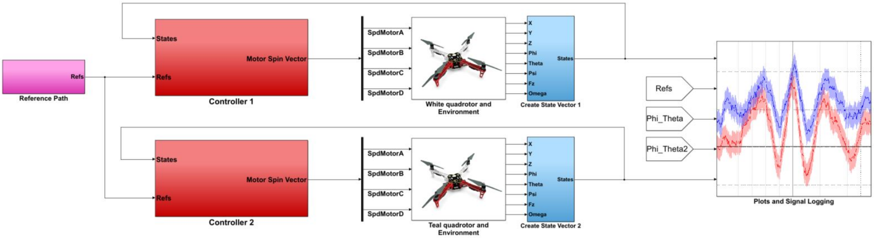
Simulator Layout



Simulator Layout

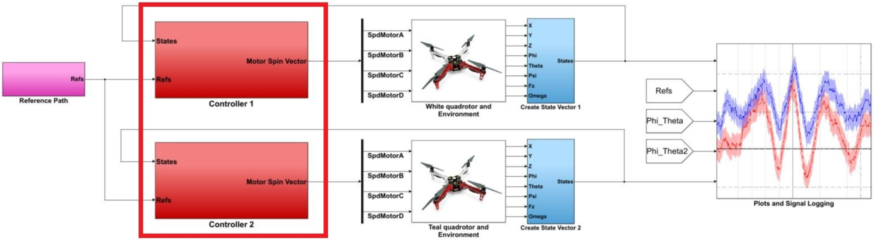


Simulator Layout II



- Reference trajectory generation;
- Control algorithms;
- Vehicle dynamics;
- Integrate differential equations and add noise

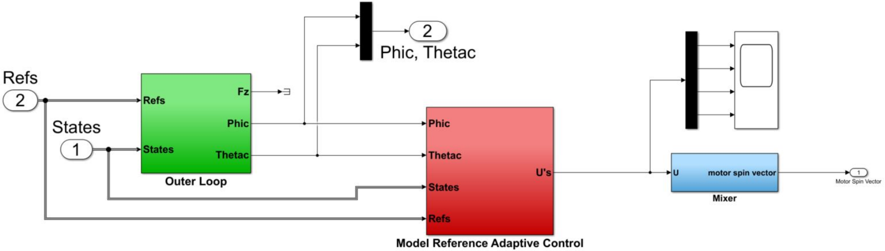
Simulator Layout II



- Reference trajectory generation;
- Control algorithms;
- Vehicle dynamics;
- Integrate differential equations and add noise

Simulator Layout III

Control blocks all have the same layout:



- Outer loop control;
- Inner loop control;
- Mixer

Tips for Tuning MRAC

Reference Model

- Design frequency response of reference model to match the real system;
- For quadcopters, the outer loop is slower than the inner loop;

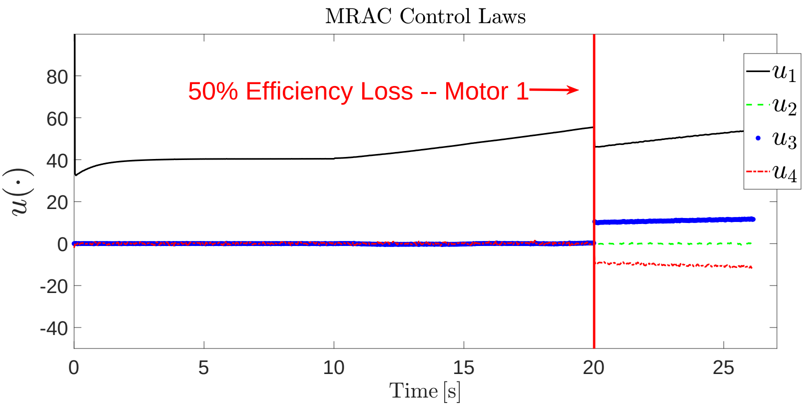
Tuning the Adaptive Rates

- Start small: $\Gamma \approx 1 - 10$;
- Increase rates until oscillations begin, then decrease;
- In our experience:
  - Γ_x is most sensitive;
  - Higher Γ_r = better command tracking;
  - Higher Γ_x, Γ_Θ = better disturbance rejection

MRAC vs. PID

- White quadrotor – MRAC;
- Black quadrotor – PID;
- Mission: Takeoff to $1m$, follow square trajectory $4m$ sides;
- Both experience 50% efficiency loss in the front motor (red arm)

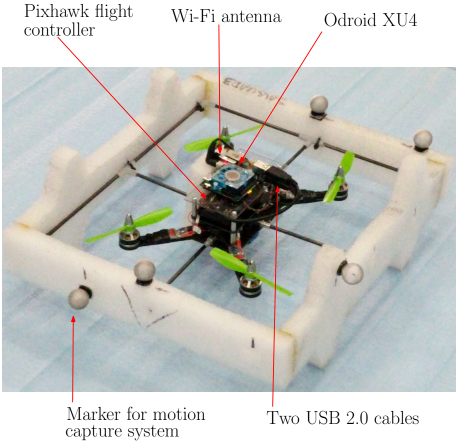
MRAC Controls Plot



MRAC controls change rapidly at $t = 20s$ to handle the motor failure disturbance

Quadcopter Architecture

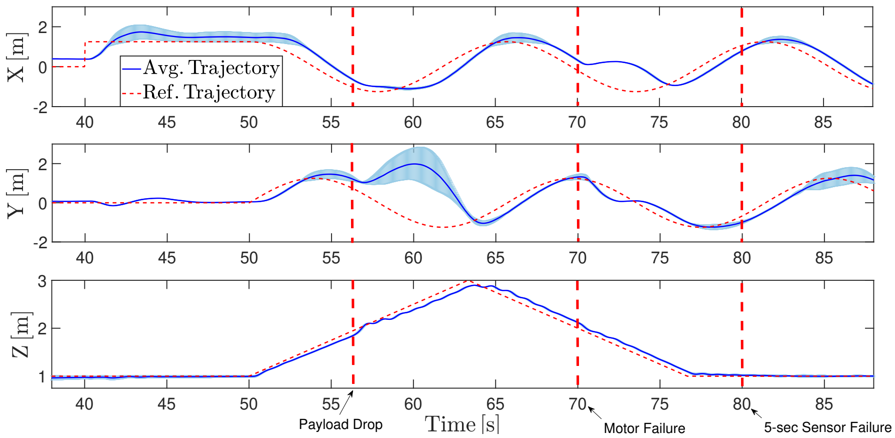
- Pixhawk flight controller used for inertial measurement unit and actuation
- ODroid XU4 companion computer to calculate controls
- QGroundControl to communicate with Pixhawk
- VICON motion capture system to deduce UAV position



Mission Scenario

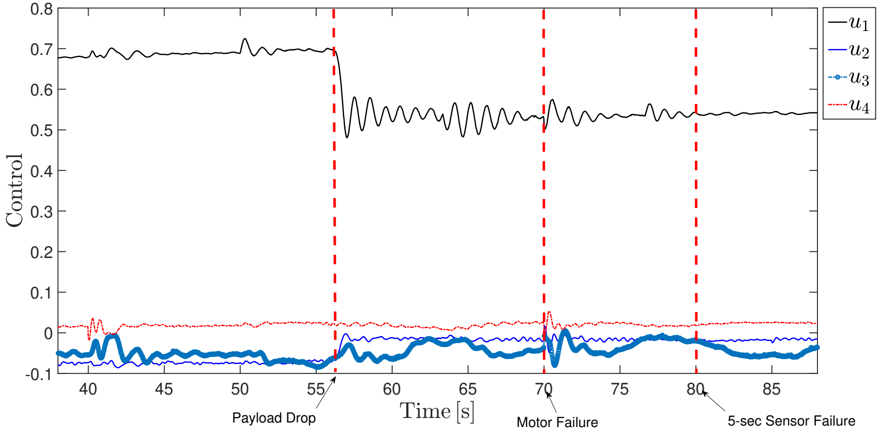
- Carry a 200g slung payload,
- Takeoff to 1m height,
- Move forward 1m in X,
- Follow a spiral shaped trajectory,
- Slung payload is dropped at $t = 57\text{s}$,
- 25% motor failure of one motor at $t = 70\text{s}$,
- Roll angle frozen at $t = 80\text{s}$ for 5 seconds,
- Maintain $\psi(t) = 0, t \geq 0$,
- Land at takeoff point.

MRAC Trajectory Plot



Trajectory tracking deviates greatly when the system is disturbed, yet MRAC drives the disturbed system to the desired trajectory

MRAC Controls Plot



Controls drop to handle payload drop at $t = 57\,\mathrm{s}$.
Controls react to compensate for motor failure at time $t = 70\,\mathrm{s}$

Taguchi Method

Problem

- No known, systematic method to tune nonlinear controllers

Approach

- Input: Independent variables to test & response variable to measure
- Output: Values of independent variables that will minimize the response variable
- Exploit experimenter's familiarity with the system& analysis of variation

Taguchi Method Steps

1. Determine number of independent variables I
   - Up to the experimenter's discretion
2. Determine response variable $\bar{e}_i \triangleq \frac{1}{N} \sum_{l_i=1}^{N} e_i(t_{l_i})$
3. Determine number of level settings k, per independent variable
4. Determine minimum number of experiments $E \triangleq I(k-1)+1$
5. Select orthogonal array
6. Run experiments and record response variable
7. Perform analysis of variance on response variable for each independent variable

Analysis of Variance (ANOVA)

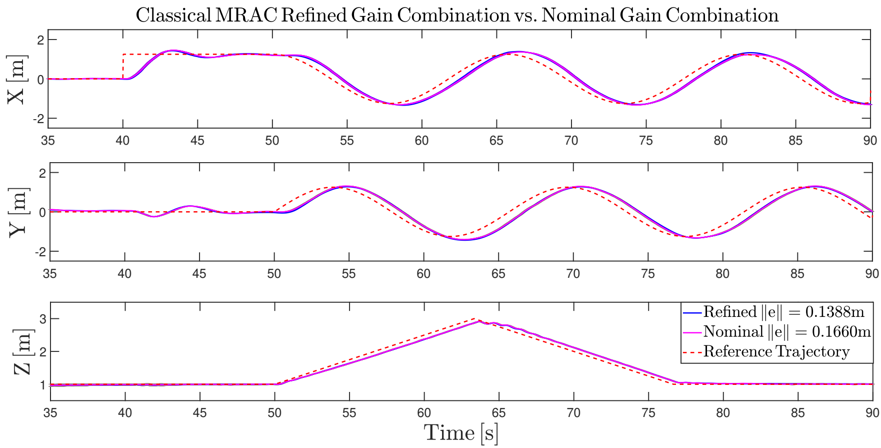
- Signal-to-noise ratio (S/N): quantify how response variable is affected by noise.
- S/N calculated for each independent variable & level setting
  - $S/N_{i,j} = -10\log(\frac{1}{n}\sum_{m=1}^{n}\overline{e}_{m,j}^2), \quad i = 1,\ldots,I, \quad j = 1,\ldots,k,$ where n is the number of times a level setting is tested
- Level setting with lowest S/N ratio selected for that independent variable
- Change in S/N indicates most sensitive independent variable

Results of Taguchi Method

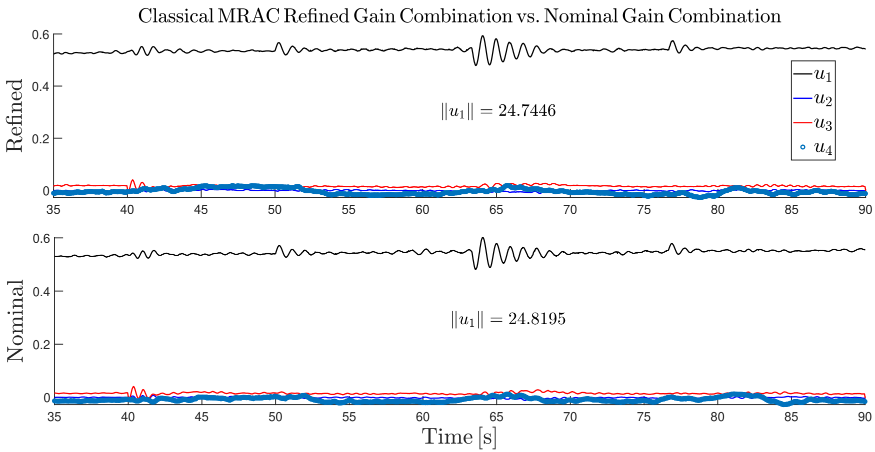
- Classical MRAC & e-MRAC laws tuned using trial & error method
  - These gains dubbed the "nominal gains"
- **Result**: "refined gain combination" & most sensitive i. variables

	Battery	MRAC	e-MRAC
Nominal $\bar{\bar{e}}$ (meters)	Fresh (16.5V)	0.1771	0.1958
	Med (15.9V)	0.1736	0.1812
	Low (15.1V)	0.1474	0.1650
	Average	0.1660	0.1807
Refined $\bar{\bar{e}}$ (meters)	Fresh (16.5V)	0.1397	0.1374
	Med (15.9V)	0.1383	0.1424
	Low (15.1V)	0.1385	0.1290
	Average	0.1388	0.1363
	% Change	-16.36	-24.58

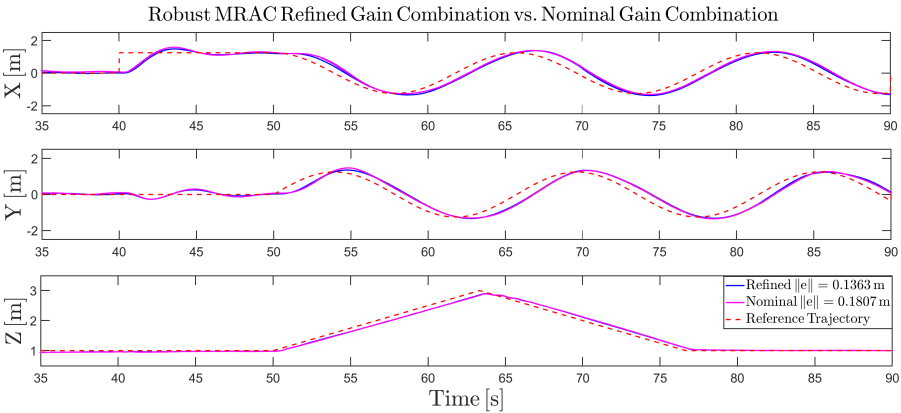
MRAC Trajectory Plot



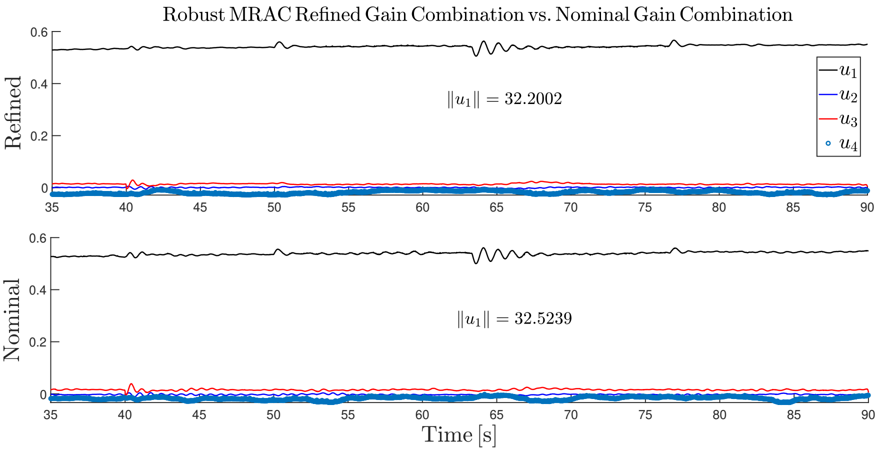
MRAC Controls Plot



e-MRAC Trajectory Plot



e-MRAC Controls Plot



Questions?

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