Theory and Algorithms for Safe and Resilient Multi-Agents Systems

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Joint work with

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Aerospace Engineering, University of Michigan

Multi-Agent Planning and Control

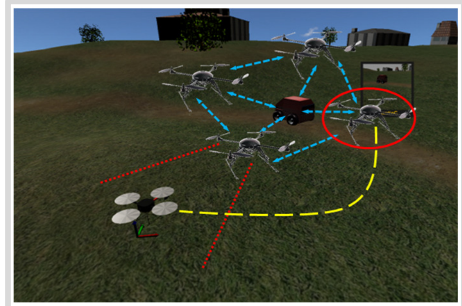
Ground, marine, aerial, space vehicles

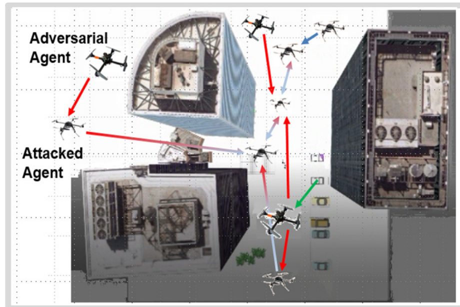
Safety and Resilience under Uncertainty

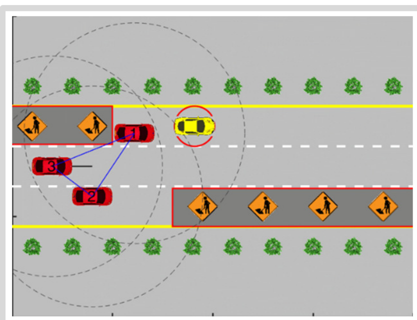
Towards advancing autonomy

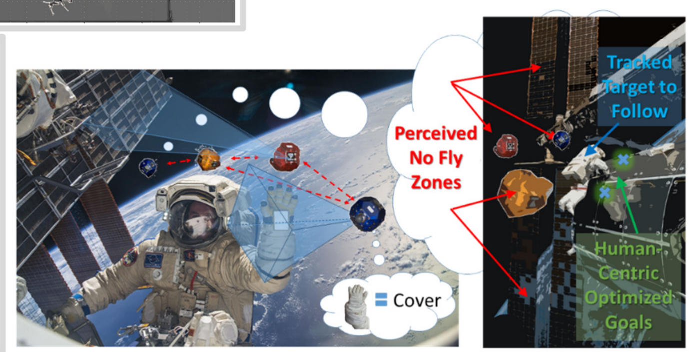
Nonlinear Control and Estimation

Robust control, estimation and learning

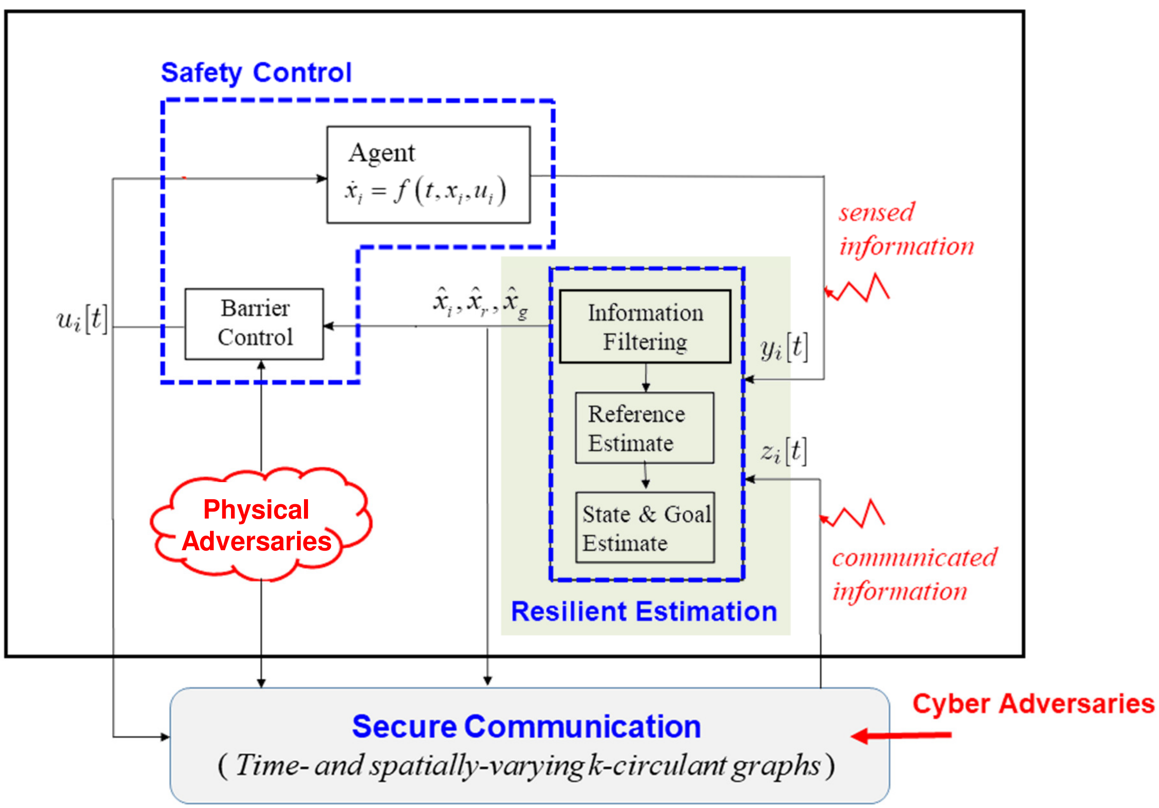


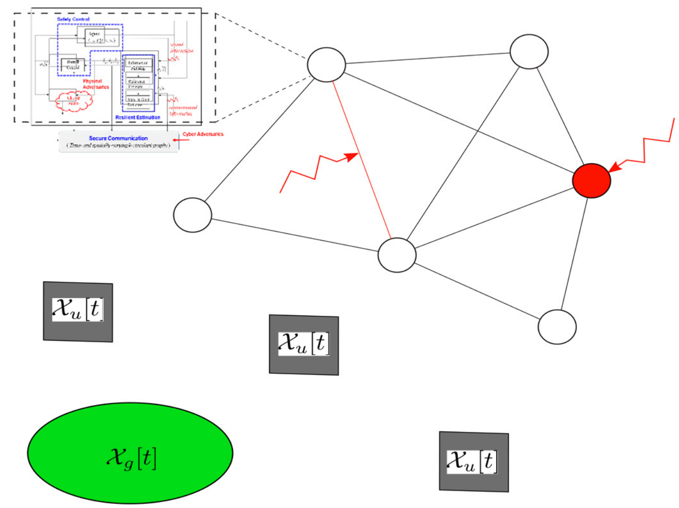






Safety and Resilience Architecture



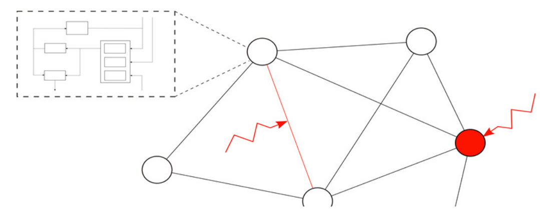


Outline

- Resilient Multi-Agent Networks
    - Information Reconstruction
    - Formation Control
- Safety Control under Spatiotemporal Constraints
    - Finite-Time Stability (FTS) and Fixed-Time Stability (FxTS)
    - Fixed-Time Control Lyapunov Functions
        - QP approach
        - CLF approach (WeB18.5)
- Future Research

Earlier Resilience Results

- Network as a digraph $\mathcal{G} = (\mathcal{V}, \mathcal{E})$ $\quad \mathcal{V} = \{1, \ldots, n\}$ $\quad \mathcal{E} \subseteq \mathcal{V} \times \mathcal{V}$
- Up to F-local adversaries
  - x Share malicious information and/or do not play consensus



- Resilient Communication Graphs
  - r-robustness and (r,s)-robustness

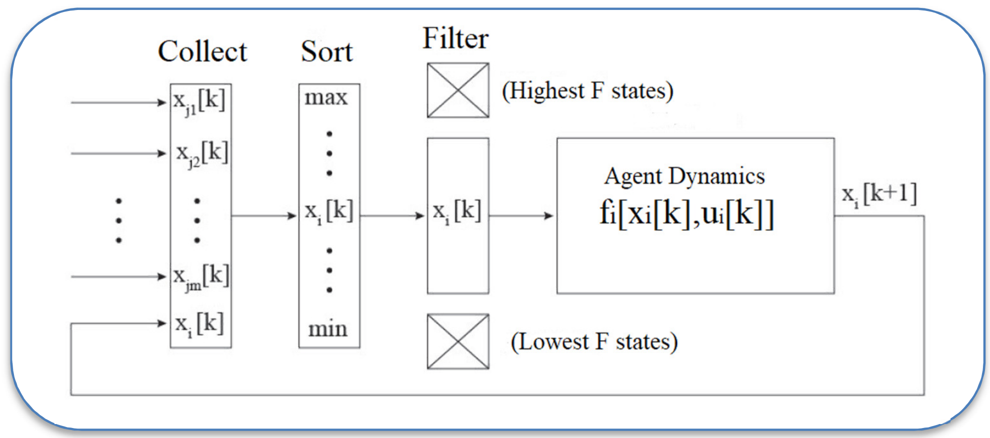
Definition 1

A set $S \subset \mathcal{V}$ is r-reachable ($r \in \mathbb{Z}_{\geq 0}$) if $\exists i \in S$ such that $|\mathcal{V}_i \backslash S| \geq r$

Definition 2

A digraph $\mathcal{G}$ is r-robust if for all nonempty, disjoint $S_1, S_2 \subset \mathcal{V}$, at least one subset is r-reachable.

- Resilient Filtering: W-MSR algorithm
- Principle: Each agent
  - sorts received information
  - filters out the F highest and F lowest values
- Consensus if the network is
  - (2F+1)-robust or (F+1,F+1)-robust



- Challenges:
- Checking r-robustness and (r,s)-robustness is NP-hard
- Consensus to arbitrary reference values is not guaranteed

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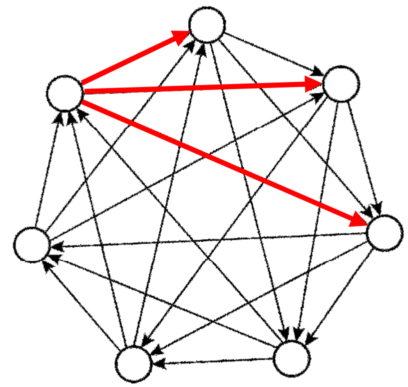
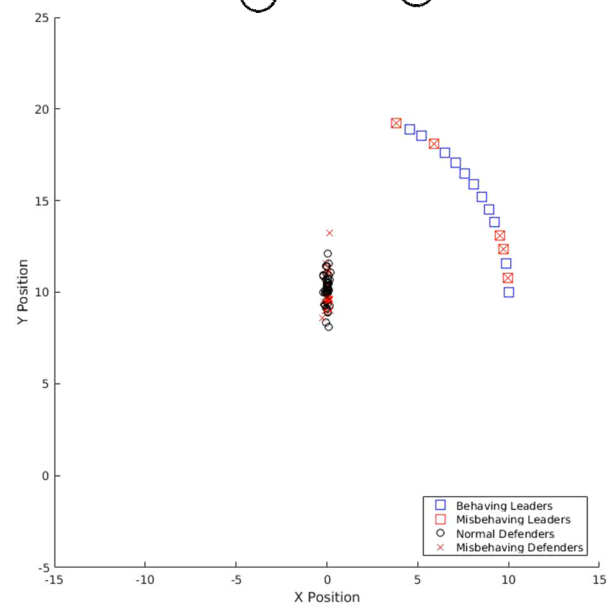
Our Resilience Results

- [1]: *k*-circulant graphs have r-robustness and (r,s)-robustness as functions of *k*
  - Resilient, scalable network topologies [**CDC17**]
- [2]: Resilient consensus to **arbitrary** reference values in time-invariant and time-varying graphs
  - Resilient Leader-Follower consensus [**ACC18**]
- [3]: **Resilient formation control**
  - In **finite** time under **bounded** control inputs [**CDC18**]
- [4]: Graph r-robustness and (r,s)-robustness as a MILP
  - More efficient than state-of-the-art methods [**ACC19**]
  - Approximate lower bounds of r- and (r,s)-robustness
- [5]: Resilient Barriers for Undirected Networks

- J. Usevitch et. al. (Journal versions: **[5], [6], [7]**)

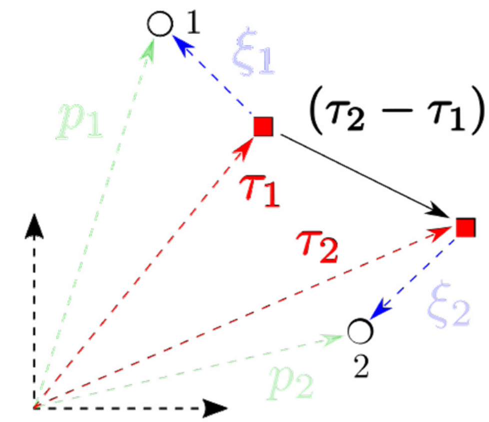


3-circulant graph



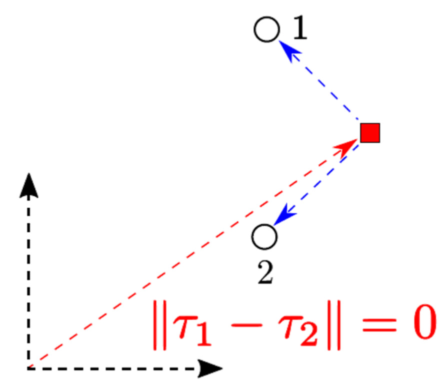
Resilient Formations: Problem Statement

- Time invariant digraph $\mathcal{D} = (\mathcal{V}, \mathcal{E})$, $\mathcal{V} = \{1, \ldots, n\}$
- Agent states $\boldsymbol{p}_i \in \mathbb{R}^n, i \in \mathcal{V}$



- $\boldsymbol{\xi}_i \in \mathbb{R}^n \ \forall i \in \mathcal{V}$: Formation vectors (target locations)
- $\boldsymbol{\tau}_i = \boldsymbol{p}_i(t) - \boldsymbol{\xi}_i \ \forall i \in \mathcal{V}$: Center of formation

- How can the formation be achieved in the presence of misbehaving agents?
- What are the communication topologies and information filters that ensure resilient consensus?



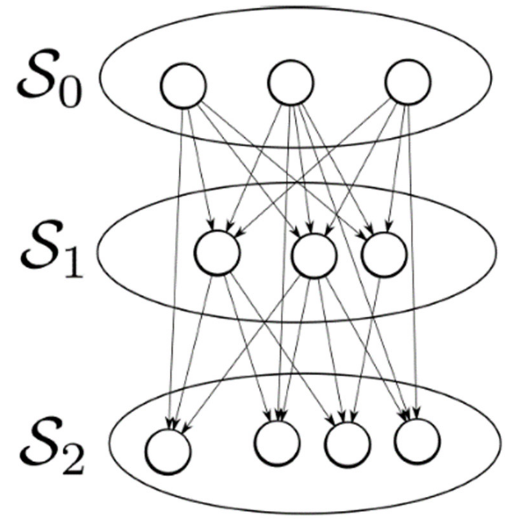
Resilient Formations: Communication Topology

Definition 1 (Resilient Directed Acyclic Graph (RDAG))

Digraph $\mathcal{D} = (\mathcal{V}, \mathcal{E})$ *is RDAG with parameter* $r \in \mathbb{N}$ *if all of the following properties hold:*

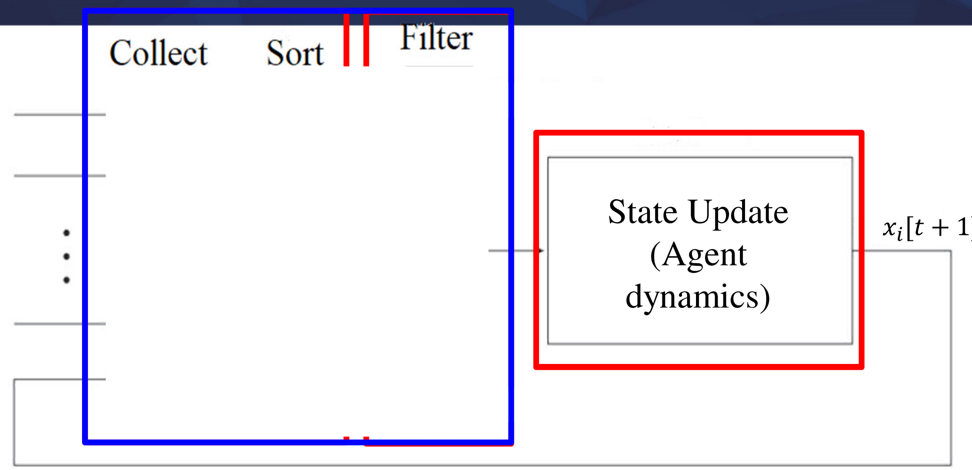
1. *There exists partitioning of* $\mathcal{V}$ *into* $\mathcal{S}_0, \ldots, \mathcal{S}_m \subset \mathcal{V},\ m \in \mathbb{Z}$ *such that* $|\mathcal{S}_0| \geq r$
2. *For each* $i \in \mathcal{S}_j,\ 1 \leq j \leq m,$ $\mathcal{V}_i \subseteq \bigcup_{k=0}^{j-1} S_k$
3. *For each* $i \in \mathcal{S}_j,\ 1 \leq j \leq m,$ $|\mathcal{V}_i| \geq r$

1) The size of the layer S_0 is at least r
2) In-neighbors are only from layers above
3) Each agent has at least r in-neighbors



Resilient Formations: Information Filtering

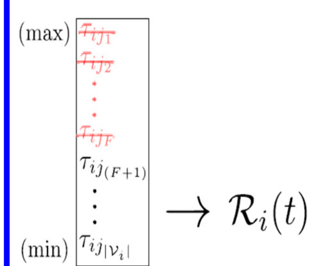
1D W-MSR Filtering



The norm-based W-MSR filtering modifies the above version by collecting and sorting the neighbors' normed information from highest to lowest value, and removing only the F highest values

Algorithm 1 Continuous-Time Filtering

procedure UPDATEFILTEREDLIST
  Calculate $\tau_{ij} = \|\boldsymbol{\tau}_j - \boldsymbol{\tau}_i\| \;\; \forall j \in \mathcal{V}_i$
  if $t = m\epsilon_d, \; m \in \mathbb{Z}_{\geq 0}, \; \epsilon_d > 0$ **then**
    Sort τ_{ij} values such that $\tau_{ij_1} \geq \ldots \geq \tau_{ij_{|\mathcal{V}_i|}}$
    $\mathcal{R}_i(t) \leftarrow \{j : \tau_{ij} \in \{\tau_{ij_{F+1}}, \ldots, \tau_{ij_{|\mathcal{V}_i|}}\}\}$



- $\mathcal{R}_i(t)$: Filtered list after removing F maximum elements
- Update on discrete instances $t = m\epsilon_d$

Resilient Formations: Finite-Time Controller

Closed loop system:

$$\dot{\boldsymbol{\tau}}_i = \boldsymbol{u}_i,$$
$$\boldsymbol{u}_i(t) = \gamma_i(t) \sum_{j \in \mathcal{R}_i(t)} w_{ij}(t)(\boldsymbol{\tau}_j - \boldsymbol{\tau}_i)\|\boldsymbol{\tau}_j - \boldsymbol{\tau}_i\|^{\alpha-1}, \ 0 < \alpha < 1$$

where

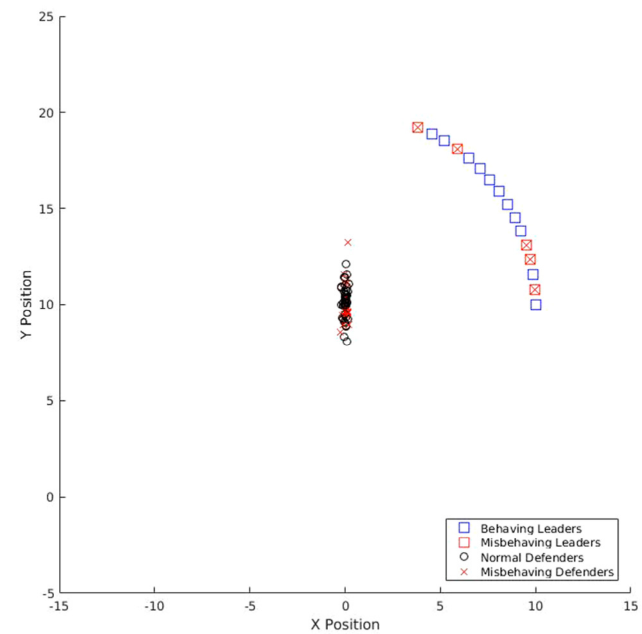
- $\gamma_i(t) = \frac{\sigma_i(t)}{\|\boldsymbol{u}_i^p\|}$
- Saturation function:

$$\sigma_i(t) = \min\{\|\boldsymbol{u}_i^p(t)\|, u_M\},$$
$$\boldsymbol{u}_i^p(t) = \sum_{j \in \mathcal{R}_i(t)} w_{ij}(t)(\boldsymbol{\tau}_j(t) - \boldsymbol{\tau}_i(t))\|\boldsymbol{\tau}_j - \boldsymbol{\tau}_i\|^{\alpha-1}, \ 0 < \alpha < 1$$

- Input satisfies bounds $\|\boldsymbol{u}_i\| \leq u_M \ \forall i \in \mathcal{V}$

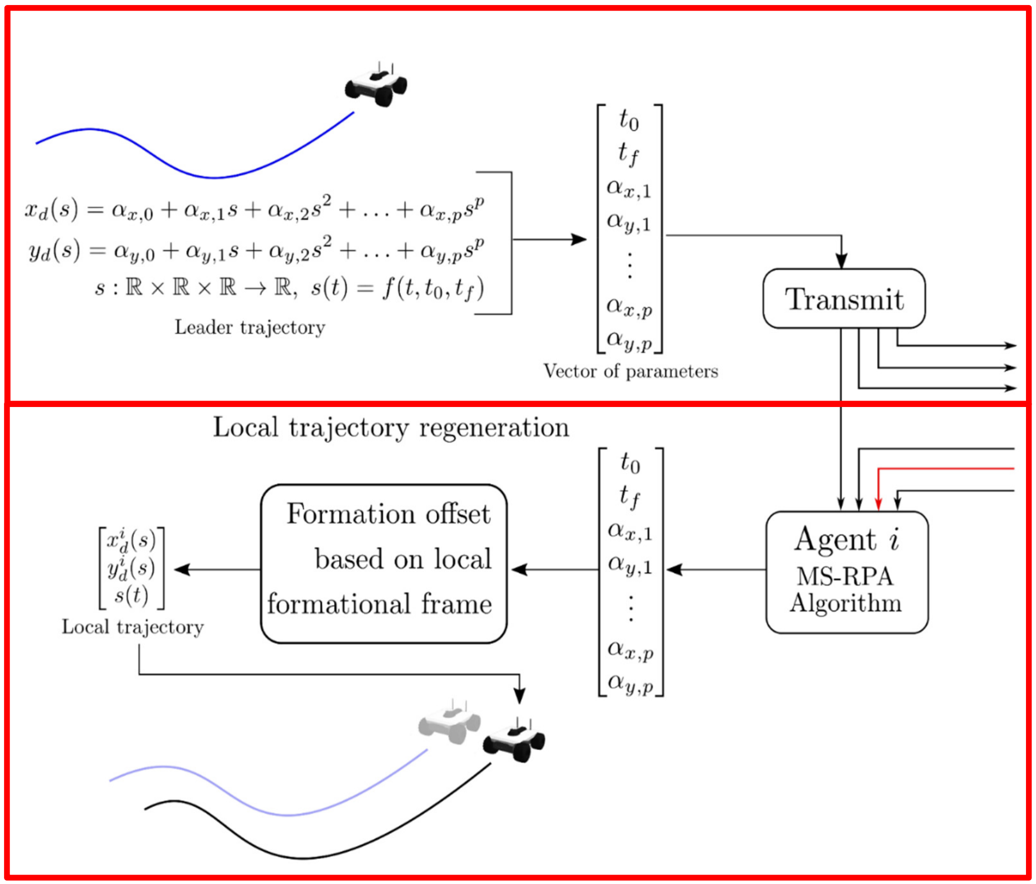
Theorem 2

Consider a digraph $\mathcal{D}$ which is an RDAG with parameter $3F+1$, where $\mathcal{S}_0 = \mathcal{L}$ and $\mathcal{A}$ is an F-local set. Under the proposed closed loop dynamics, $\boldsymbol{\tau}_i$ will converge to $\boldsymbol{\tau}_L$ in finite time for all normal agents $i \in \mathcal{N}$.



RDAG of 80 agents
r = 16
F = 5 local model
m = 5 sublevels

Resilient Formations: System Architecture



Leaders:

- Determine trajectory for center of formation (COF)
- Encode COF trajectory into unique parameters
- Resiliently transmit parameters to out-neighbors

Followers:

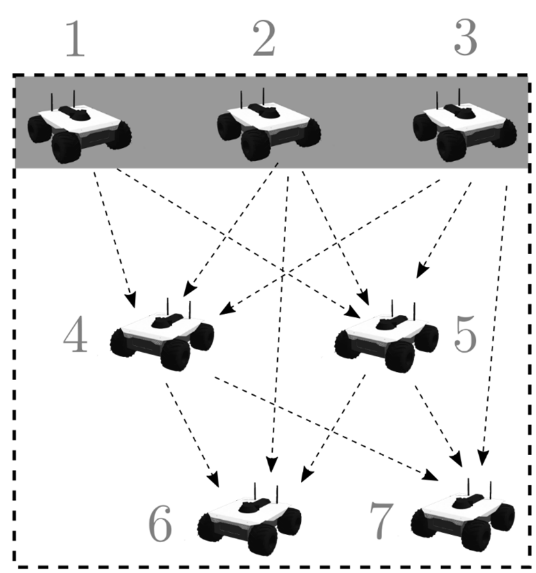
- Receive and accept parameters only if resilience criteria satisfied
- Reconstruct unique trajectory of COF
- Add local formation offset to obtain local desired trajectory
- Track local trajectory

Resilient Formations: Information Propagation

Multi-Source Resilient Propagation Algorithm [8]

- RDAG with parameter (2F+1)
- F-local misbehaving agent model
- Including misbehaving leaders
- S_0 layer comprises of leaders only

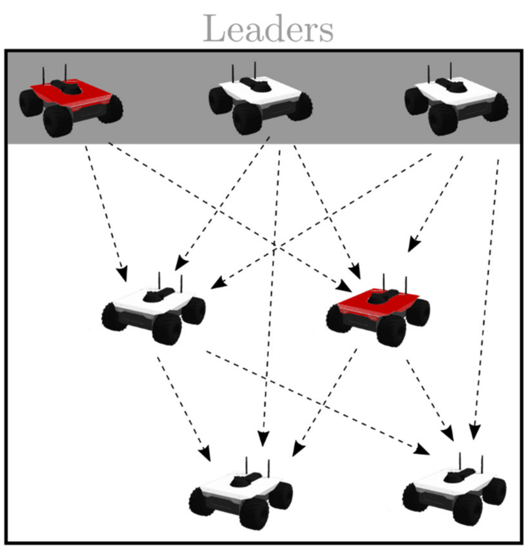
- Example: RDAG with r=3



Resilient Formations: Information Propagation

Multi-Source Resilient Propagation Algorithm [8]

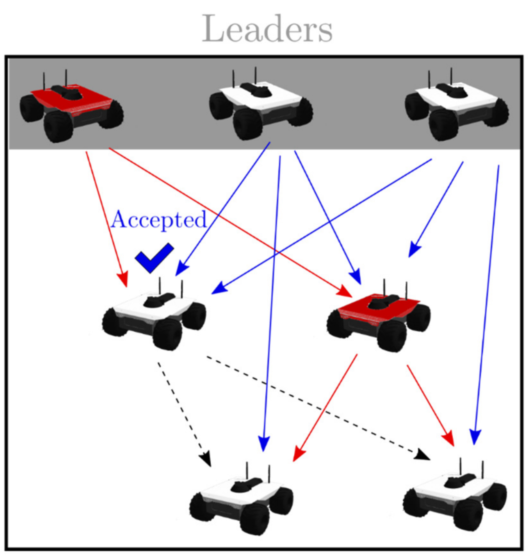
- Leaders transmit message to out-neighbors



Resilient Formations: Information Propagation

Multi-Source Resilient Propagation Algorithm [8]

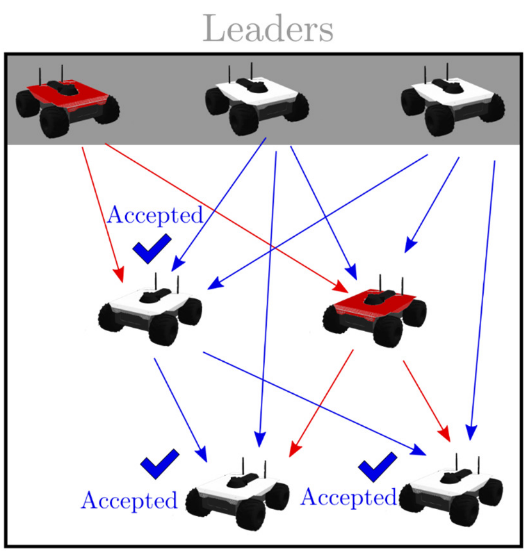
- Leaders transmit message to out-neighbors
- Followers accept message if identically received from at least (F+1) in-neighbors



Resilient Formations: Information Propagation

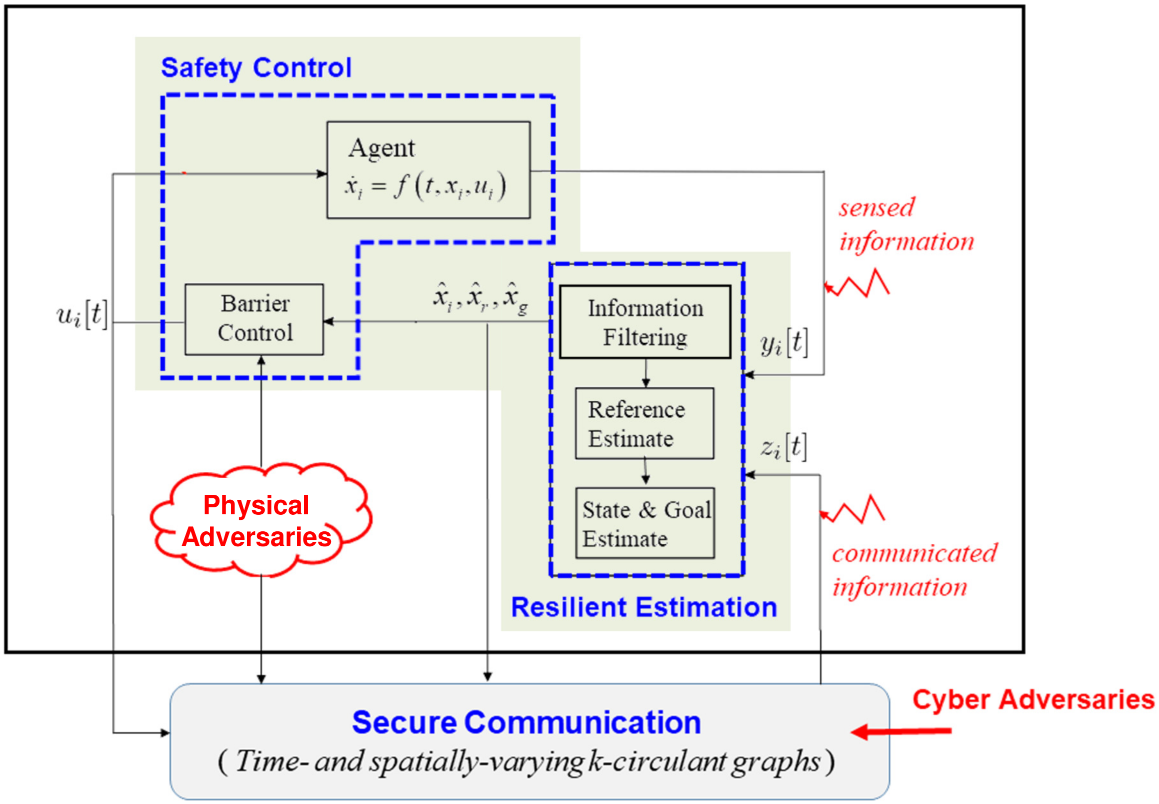
Multi-Source Resilient Propagation Algorithm [8]

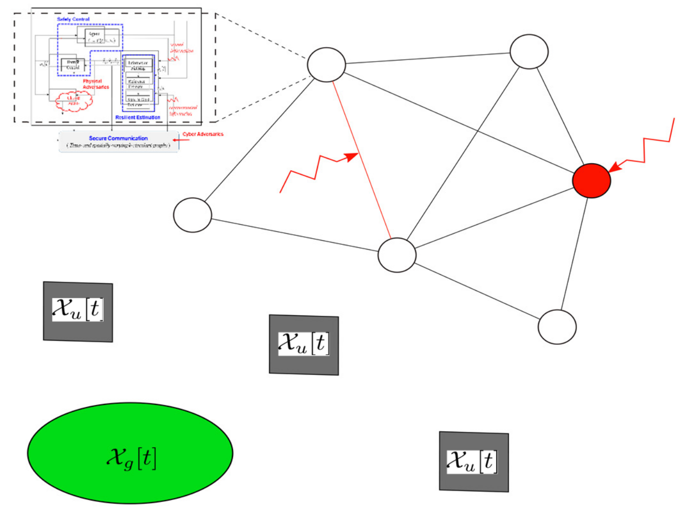
- Leaders transmit message to out-neighbors
- Followers accept message if identically received from at least (F+1) in-neighbors
- Accepted messages by followers transmitted to their out-neighbors, and so on



Experimental Results

Safety and Resilience Architecture





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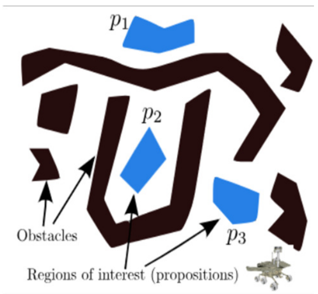
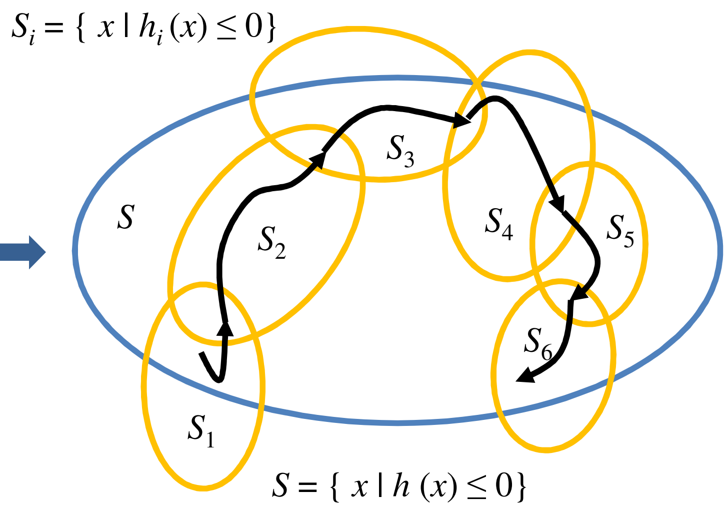
Spatiotemporal Control Synthesis: Overview



Example:
Multi-Robot
Mission Planning

LTL/STL Specification
+
$\dot{x}_j = f_j(x_j, u_j)$
$u_j \in U$ $x_j \in X_s$

Multi-Agent



Single-Agent

○ **Safety (set invariance)**
State trajectories must remain in a safe set

○ **Performance (set attractivity)**
State trajectories must reach desired sets within **specified** time intervals

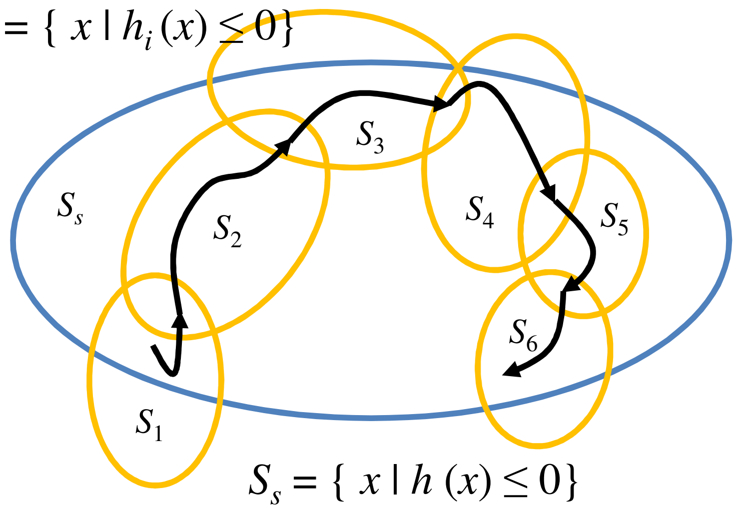
Spatiotemporal Control: Approach

- Synthesis tools:
  - Quadratic Programs (QPs) for FTS/FxTS/PTS **[9, 10]**
  - Modified Sontag's Formula for PTS (**ACC20 Paper WeB18.5**) **[11]**
- Analysis tools:
  - FTS of Switched/Hybrid Systems **[12]**

- **K. Garg, E. Arabi, and D. Panagou**

Spatiotemporal Control Synthesis via QP

Let $\dot{x} = f(x) + g(x)u$ where $x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$



Assume that:

- There exists a safe set $S_s = \{x \in \mathbb{R}^n \mid h(x) \leq 0\}$ where $h(x)$ is continuously differentiable
- There exist sets $S_i = \{x \in \mathbb{R}^n \mid h_i(x) \leq 0\}, i \in \{0, 1, \ldots, N\}$ where $h_i(x)$ are continuously differentiable
- $S_s \cap S_0 \neq \emptyset, S_i \cap S_{i+1} \neq \emptyset,$ for $0 \leq i \leq N-1$
- There exist time intervals $[t_i, t_{i+1})$ such that $t_{i+1} - t_i \geq \bar{T}$

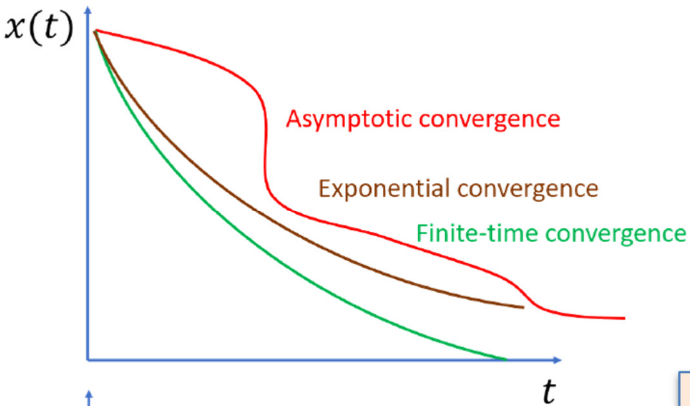
Problem statement (Problem 1)

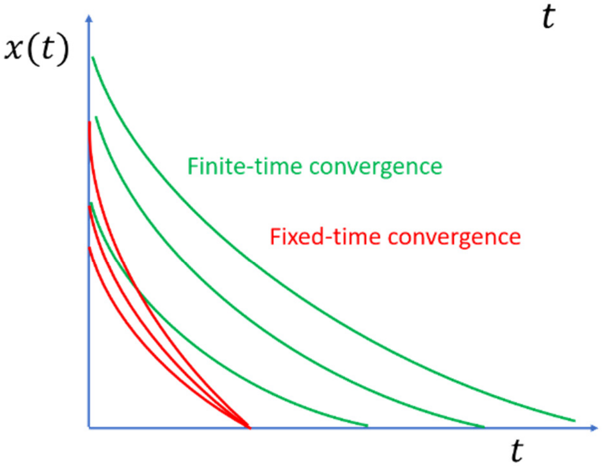
Find a control input $u(t) \in U = \{A_u u \leq b_u\}$ such that for $x(0) \in S_s \cap S_0$,

- $x(t) \in S_s$, $\forall t \geq 0$,
- $x(t) \in S_i$, $\forall t \in [t_i, t_{i+1})$

Finite-Time and Fixed-Time Stability

Let $\dot{x} = f(x)$
where f is continuous, $f(0) = 0$





Finite-time Stability (FTS) (Bhat and Bernstein, 2000)

Theorem 1. *Suppose there exists a positive definite function V for system (1) such that*

$$\dot{V}(x) \leq -cV(x)^{\beta},$$

with $c > 0$ and $0 < \beta < 1$. Then, the origin of (1) is FTS with settling time function

$$T(x(0)) \leq \frac{V(x(0))^{1-\beta}}{c(1-\beta)}.$$

Fixed-time Stability (FxTS) (Polyakov, 2012)

Theorem 1 ([2]). *Suppose there exists a positive definite function V for system (1) such that*

$$\dot{V}(x) \leq -aV(x)^{p} - bV(x)^{q}$$

with $a, b > 0$, $0 < p < 1$ and $q > 1$. Then, the origin of (1) is FxTS with continuous settling time T that satisfies

$$T \leq \frac{1}{a(1-p)} + \frac{1}{b(q-1)}.$$

Prescribed-time Stability (PTS)

Time of convergence T can be chosen **arbitrarily** by the user.
Also called **predetermined** or **predefined.**

Control Barrier Functions

Reciprocal Control Barrier Functions (Ames et al, TAC 2017)

Definition: Let $\dot{x} = f(x) + g(x)u$, where $f(x), g(x)$ are locally Lipschitz

$x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$

A continuously differentiable function $B : \text{Int}(\mathcal{C}) \to \mathbb{R}$ is called a Reciprocal Control Barrier Function (RCBF) for the set C if there exist class K functions $\alpha_1, \alpha_2, \alpha_3$ such that for all $x \in \text{Int}(\mathcal{C})$

$$\frac{1}{\alpha_1(h(x))} \leq B(x) \leq \frac{1}{\alpha_2(h(x))}$$

$$\inf_{u \in U} [L_f B(x) + L_g B(x)u - a_3(h(x))] \leq 0$$

Let the set $K_{rcbf}(x) = \{u \in U : L_f B(x) + L_g B(x)u - a_3(h(x)) \leq 0\}$

Then any locally Lipschitz $u : \text{Int}(\mathcal{C}) \to U$ such that $u(x) \in K_{rcbf}(x)$ will render Int(C) a forward invariant set.

CLF-CBF QPs

Let the following CLF-CBF QP

$$\mathbf{u}^\star(x) = \arg \min_{\mathbf{u}=(u,\delta)\in\mathbb{R}^m\times\mathbb{R}} \frac{1}{2}\mathbf{u}^T H(x)\mathbf{u} + F(x)^T\mathbf{u}$$

s.t.

$$L_f V(x) + L_g V(x) u + c_3 V(x) - \delta \leq 0$$
$$L_f B(x) + L_g B(x) u - \alpha(h(x)) \leq 0$$

Theorem [Ames et al, TAC 2017]:

Suppose that:

the vector fields f and g of the control system,

the gradients of the RCBF B and CLF V,

the cost function terms $H(x)$ and $F(x)$ in (CLF-CBF QP)

are all locally Lipschitz. Suppose furthermore that

$L_g B(x) = 0$ for all $x \in \text{Int}(C)$.

Then the solution, $\mathbf{u}^*(x)$, of (CLF-CBF QP) is locally Lipschitz continuous for $x \in \text{Int}(C)$. Moreover, a closed-form expression can be given for $\mathbf{u}^*(x)$.

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- Spatiotemporal Control Synthesis
  - Finite-Time Stability (FTS) and Fixed-Time Stability (FxTS)
  - Fixed-Time Control Lyapunov Functions
    - QP approach
    - CLF approach (WeB18.5)

- Future Research

Fixed-Time Control Lyapunov Function

Let $\dot{x} = f(x) + g(x)u$ where $x \in \mathbb{R}^n, u \in U \subset \mathbb{R}^m$

Definition: The continuously differentiable function $V : \mathbb{R}^n \to \mathbb{R}$ is called a **Fixed-Time Control Lyapunov Function** wrt a set S (FxT-CLF-S) of the system with parameters a_1, a_2, b_1, b_2 if

i) It is positive definite wrt a closed set S, i.e.,

$$V(x) > 0 \text{ for } x \notin S$$
$$V(x) = 0 \text{ for } x \in \partial S$$

ii) $\inf_u [L_f V(x) + L_g V(x)u] \leq -a_1 (V(x))^{b_1} - a_2 (V(x))^{b_2}, \ \forall x \notin \text{Int}(S)$

where $a_1, a_2 > 0, \ b_1 > 1, \ 0 < b_2 < 1$ satisfy $\dfrac{1}{a_1(b_1 - 1)} + \dfrac{1}{a_2(1 - b_2)} \leq \bar{T}$ with $\bar{T}$ being a user-defined time.

FxT-CLF-CBF for Spatiotemporal Control

Theorem [9]

If there exist $a_{i1}, a_{i2}, \lambda, \lambda_i > 0$, $b_{i1} > 1, 0 < b_{i2} < 1$ and control input u such that

$$\bar{T} \geq \max_{i\in\Sigma}\left\{\frac{1}{a_{i1}(b_{i1}-1)} + \frac{1}{a_{i2}(1-b_{i1})}\right\} \qquad (C_0)$$

$$\inf_{u\in U}\{L_f h + L_g h u + \lambda h\} \leq 0 \qquad (C_1)$$

$$\inf_{u\in U}\{L_f h_i + L_g h_i u + \lambda_i h_i\} \leq 0 \qquad (C_2)$$

$$\inf_{u\in U}\{L_f h_{i+1} + L_g h_{i+1} u\} \leq -a_{i1}\max\{0, h_{i+1}\}^{b_{i1}} - a_{i2}\max\{0, h_{i+1}\}^{b_{i2}} \qquad (C_3)$$

hold for $t \in [t_i, t_{i+1})$, then, the control input $u(t)$ solves Problem 1.

- C_0 ensures exact convergence before $t = t_{i+1}$ (FxTS for settling time $\bar{T}$)
- C_1 results into $h(x) = 0 \Rightarrow \dot{h}(x) \leq 0 \Rightarrow$ forward invariance of set S_s
- C_2 results into $h_i(x) = 0 \Rightarrow \dot{h}_i(x) \leq 0 \Rightarrow$ forward invariance of set S_i
- C_3 results into $\dot{h}_{i+1} \leq -a_{i1}h_{i+1}^{b_{i1}} - a_{i2}h_{i+1}^{b_{i2}} \Rightarrow$ FxTS to set S_{i+1}
- C_3 also results into forward invariance of S_{i+1} once $x(t) \in S_{i+1}$

FxT-CLF-CBF for Spatiotemporal Control

A Quadratic Program (QP) to solve Problem 1

Theorem [9]

Let the solution to the following QP defined for $t \in [t_i, t_{i+1})$:

$$\min_{v, a_{i1}, a_{i2}, \lambda_i, \delta} \frac{1}{2} v^2$$

$$s.t. \; L_f h_i + L_g h_i v + \lambda_i h_i \leq 0,$$
$$L_f h_{i+1} + L_g h_{i+1} v \leq \delta h_{i+1} - a_{i1} \max\{0, h_{i+1}\}^{b_{i1}} - a_{i2} \max\{0, h_{i+1}\}^{b_{i2}},$$
$$A_u v \leq b_u,$$
$$\frac{2}{\bar{\bar{T}}} \leq a_{i1}(b_{i1} - 1) \leq a_{i2}(1 - b_{i2}),$$

be denoted as $[\bar{v}_i(t), a_{i1}, a_{i2}, \lambda_h, \lambda_i]$. Then, $u(t) = \bar{v}_i(t)$ for $t \in [t_i, t_{i+1})$ solves Problem 1.

Robust Fixed-Time Stability

Theorem (Robust FxTS Theorem)

Let $V : \mathbb{R}^n \to \mathbb{R}$ *be a* $\mathcal{C}^1$*, positive definite function, satisfying*

$$\dot{V} \leq -c_1 V^{a_1} - c_2 V^{a_2} + c_3 V,$$

with $c_1, c_2 > 0$, $a_1 = 1 + \frac{1}{\mu}$, $a_2 = 1 - \frac{1}{\mu}$ *for some* $\mu > 1$*, along the system trajectories. Then, there exists* $D \subset \mathbb{R}^n$ *such that for all* $x(0) \in D$*, the system trajectories reach the origin in a fixed time* T*. Furthermore, if* $c_3 < 2\sqrt{c_1 c_2}$*, and* V *is radially unbounded, then* $D = \mathbb{R}^n$*.*

- Relaxation of condition $\dot{V} \leq -c_1 V^{a_1} - c_2 V^{a_2}$
- Robustness w.r.t. additive vanishing disturbance if origin of *nominal* system is FxTS
- Helps guarantee feasibility of QP

FxT-CLF-CBF for Spatiotemporal Control

Consider the following optimization problem:

$$\min_{u,\delta_1,\delta_2} \quad \frac{1}{2}\|u\|^2 + p_1\delta_1^2 + p_2\delta_2^2 \qquad \delta_1, \delta_2 \text{ - slack terms}$$

$$\text{s.t.} \quad A_u u \le b_u, \qquad \text{Control input constraint}$$

$$L_f h_g(x) + L_g h_g(x)u \le \delta_1 h_g(x) - \alpha_1 h_g(x)^{\gamma_1} - \alpha_2 h_g(x)^{\gamma_2}, \qquad \text{PT-CLF condition for } S_g$$

$$L_f h_s(x) + L_g h_s(x)u \le -\delta_2 h_s(x), \qquad \text{ZCBF condition for } S_s$$

where $p_1, p_2 > 0, \gamma_1 = 1 + \frac{1}{\mu}$ and $\gamma_2 = 1 - \frac{1}{\mu}$ with $\mu > 1$, $\alpha_1 = \alpha_2 = \frac{\mu\pi}{2\bar{T}}$

- Slack terms $\delta_1, \delta_2 \rightarrow$ feasibility for all x
- δ_1 dictates region of convergence
- Convergence time $\le \bar{T}$

K. Garg, E. Arabi, D. Panagou "***Fixed-time control under spatiotemporal and input constraints: A QP based approach***," submitted to IEEE TAC, under revision.

FxT-CLF-CBF for Spatiotemporal Control

Theorem 5. *Let Assumption 3 hold. If the solution of (10), given as $(v^*(\cdot), \delta_1^*(\cdot), \delta_2^*(\cdot))$, satisfies*

$$\delta_1^*(x) < 2\sqrt{\alpha_1\alpha_2}, \quad \forall\, x \in S_S, \tag{11}$$

then, for all $x(0) \in S_S$, the closed-loop trajectories $x(t)$ under $u(\cdot) = v^*(\cdot)$ reach the set S_G in a fixed time, while satisfying safety requirement, i.e., $x(t) \in S_S$ for all $t \geq 0$. If (11) does not hold, then there exists $D \subset S_S$ such that for all $x(0) \in D$, the closed-loop trajectories satisfy $x(t) \in S_S$ for all $t \geq 0$ and reach the goal set S_G within a fixed time.

Assumption 3: The strict complementary slackness holds.

K. Garg, E. Arabi, D. Panagou "***Fixed-time control under spatiotemporal and input constraints: A QP based approach***," submitted to IEEE TAC, under revision.

Example: STL Mission Synthesis

Simulation Results

System Dynamics:

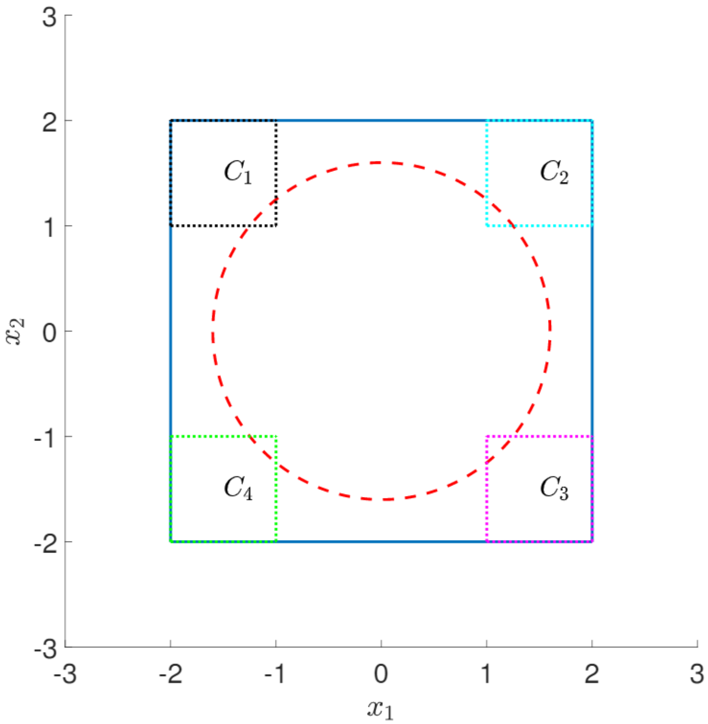
$$\dot{x}_i = u_i$$

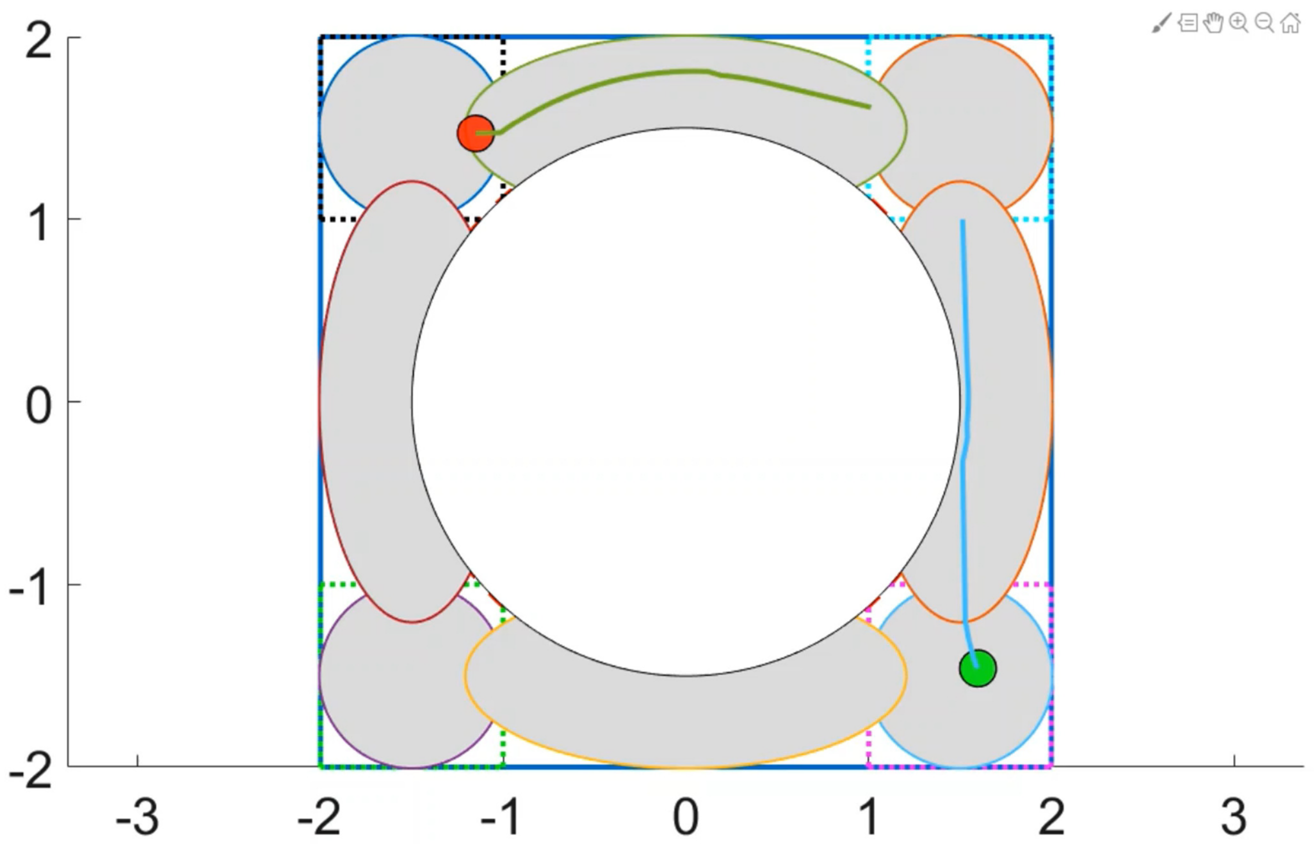
Objective:

$$(x_1, t) \vDash G_{[0,T_4]}\phi_s \wedge F_{[0,T_1]}\phi_2 \wedge F_{[T_1,T_2]}\phi_3 \wedge F_{[T_2,T_3]}\phi_4 \wedge F_{[T_3,T_4]}\phi_1$$
$$(x_2, t) \vDash G_{[0,T_4]}\phi_s \wedge F_{[0,T_1]}\phi_2 \wedge F_{[T_1,T_2]}\phi_1 \wedge F_{[T_2,T_3]}\phi_4 \wedge F_{[T_3,T_4]}\phi_3$$

Equivalently,

- $x_1(t), x_2(t) \in S_s = \left\{x_i\,(t) \middle| \, \|x_i\|_\infty \leq 2, \|x_i\|_2 \geq 1.5\right\}$ for all $t \geq 0$, and maintain a minimum separation d_m at all times
- On or before a given T_1 satisfying $0 < T_1 < \infty$, agent 1 and 2 should reach the square C_2 and so on

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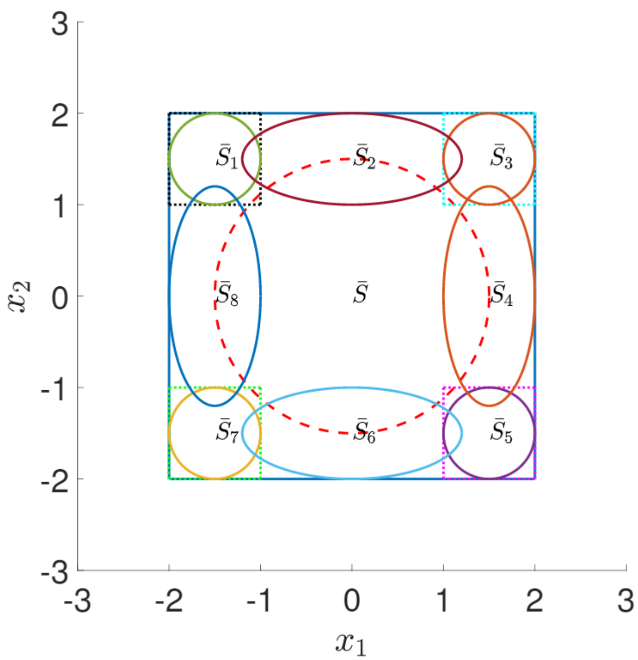
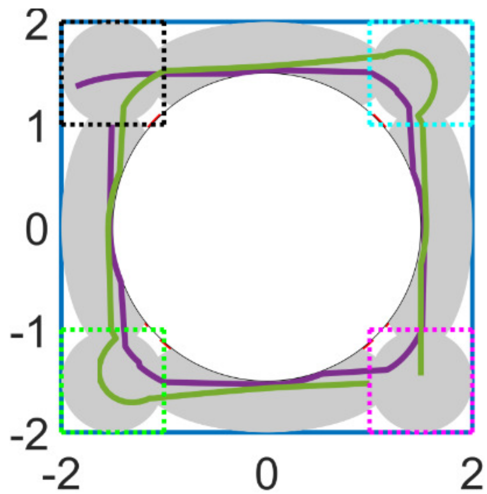
Example: STL Mission Synthesis

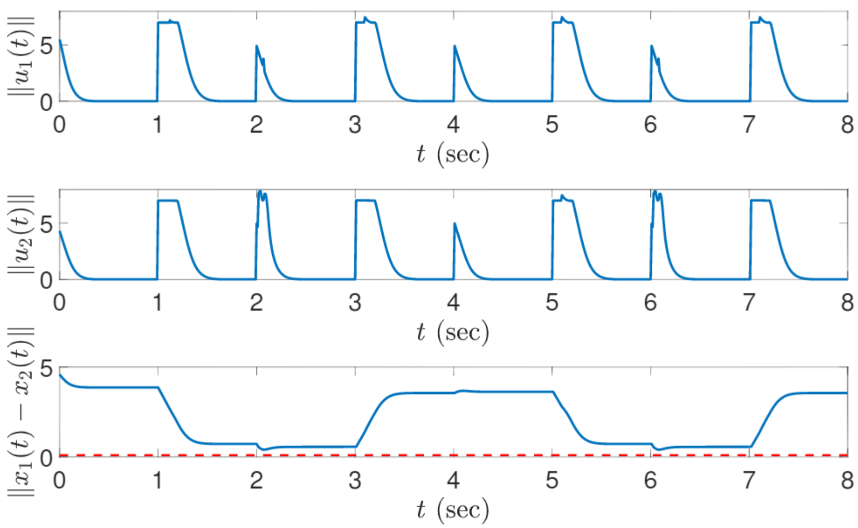
Simulation Results



Construction of sets $\bar{S}, \bar{S}_i$



Closed-loop trajectories



Control input and inter-agent distance

Outline

- Resilient Multi-Agent Networks
  - Information Reconstruction
  - Formation Control

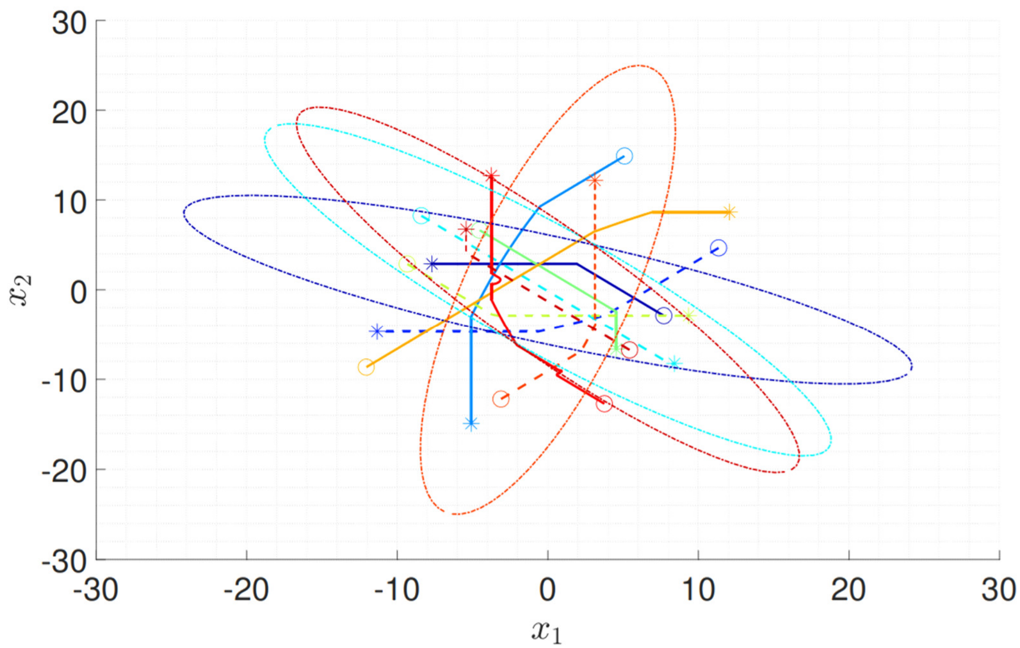
- Safety Control under Spatiotemporal Constraints
  - Finite-Time Stability (FTS) and Fixed-Time Stability (FxTS)
  - Fixed-Time Control Barrier Functions
    - QP approach
    - CLF approach (WeB18.5)

- Future Research

Multi-Agent FxT-CLF-CBFs (Ongoing Work)

Problem 1. *Find a control input* $u_i(t) \in \mathcal{U}_i = \{v \in \mathbb{R}^m; \mid u_{i,min_j} \leq v_j \leq u_{i,max_j}, j = 1, 2, \ldots, m\}$, $t \geq 0$, *such that for all* $x_i(0) \in S_{S_i}$,

- $x_i(\bar{T}) \in S_{G_i}$ *for some user-defined* $\bar{T} > 0$, *for all* $i = 1, 2, \ldots, N$;
- $\|x_i(t) - x_j(t)\| \geq d_s$, *for all* $t \geq 0$, *for all* $i \neq j$, *where* $d_s > 0$ *is a user-defined safety distance;*
- $x_i(t) \in S_{S_i}$, *for all* $t \geq 0$, *for all* $i = 1, 2, \ldots, N$.



Adversarially-Robust CBFs (Ongoing Work)

- CBF condition for set invariance

$$\sum_{i=1}^{N}\left(\frac{\partial h(\vec{x})}{\partial x_i}f_i(x_i)+\frac{\partial h(\vec{x})}{\partial x_i}g_i(x_i)u_i\right)\geq -\alpha(h(\vec{x}))$$

$\alpha:$ any locally Lipschitz extended class-$\mathcal{K}_\infty$ function

- Worst-case **adversarial** agents:

$$u_k^{\inf}(t)=\arg\inf_{u_k\in\mathcal{U}_k}\left[\frac{\partial h(\vec{x})}{\partial x_k}\left(f_k(x_k)+g_k(x_k)u_k\right)\right]$$

- Intent: drive $h(\vec{x})$ to negative value (violate set invariance)

- Best-case control action for **normal** agents:

$$u_i^{\sup}(t)=\arg\sup_{u_i\in\mathcal{U}_i}\left[\frac{\partial h(\vec{x})}{\partial x_i}\left(f_i(x_i)+g_i(x_i)u_i\right)\right]$$

- Intent: drive $h(\vec{x})$ to positive value (preserve set invariance)

$$\sum_{i\in\mathcal{V}\setminus\mathcal{A}}\sup_{u_i\in\mathcal{U}_i}\left[\frac{\partial h(\vec{x})}{\partial x_i}\left(f_i(x_i)+g_i(x_i)u_i\right)\right]+\sum_{k\in\mathcal{A}}\inf_{u_k\in\mathcal{U}_k}\left[\frac{\partial h(\vec{x})}{\partial x_k}\left(f_k(x_k)+g_k(x_k)u_k\right)\right]\geq -\alpha\left(h(\vec{x})\right)$$

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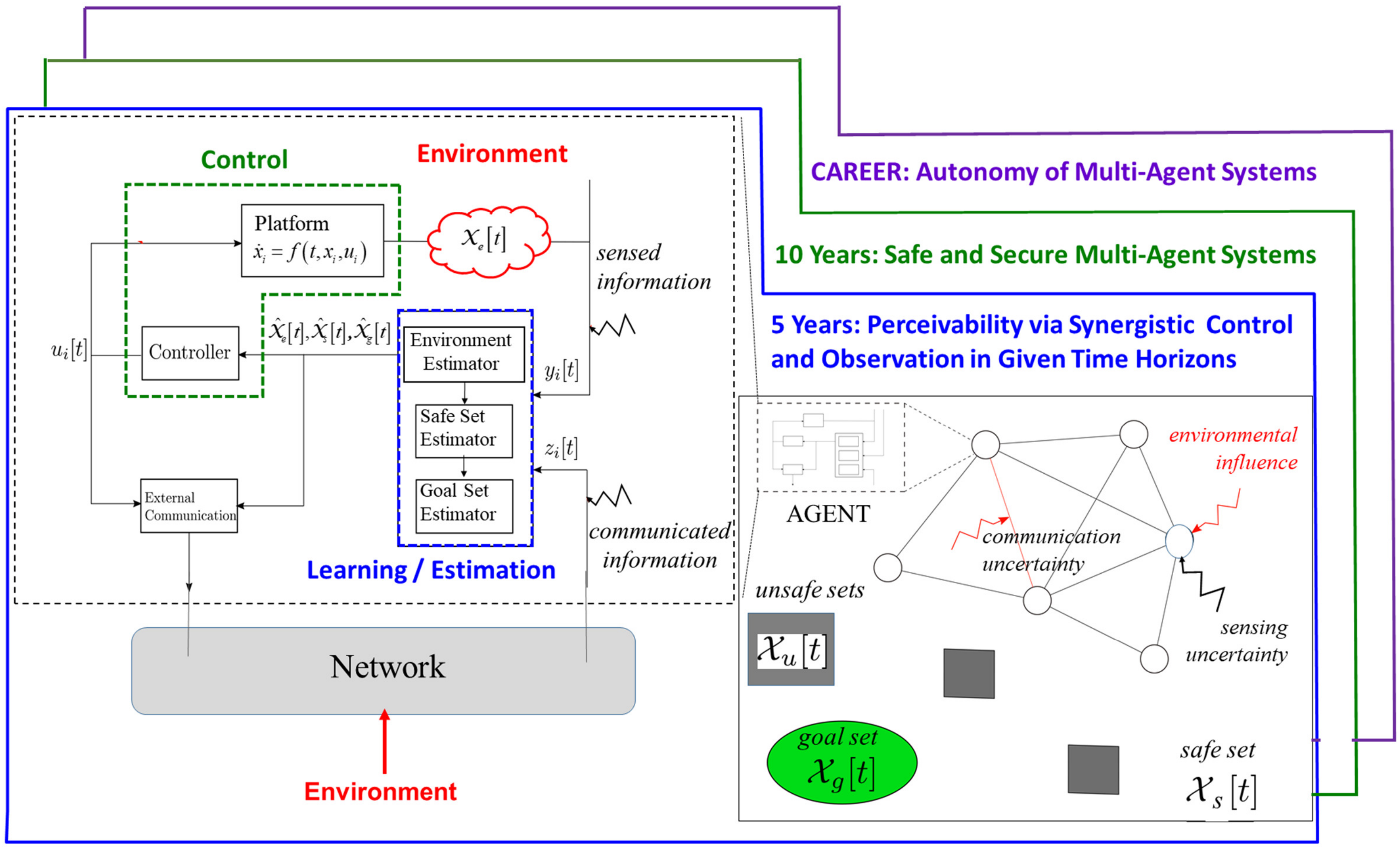

References

[1] J. Usevitch and D. Panagou "r-Robustness and (r,s)-Robustness of Circulant Graphs", CDC 2017
[2] J. Usevitch and D. Panagou, "Resilient Leader-Follower Consensus to Arbitrary Reference Values", ACC 2018
[3] J. Usevitch, K. Garg and D. Panagou "Finite-Time Resilient Formation Control with Bounded Inputs", CDC 2018
[4] J. Usevitch and D. Panagou, "Determining r-Robustness of Arbitrary Digraphs Using Zero-One Linear Integer Programming", ACC 2019
[5] J. Usevitch and D. Panagou, "Resilient Trajectory Propagation in Multi-Robot Networks," submitted to IEEE TRO, under review
[6] J. Usevitch and D. Panagou "Resilient Leader-Follower Consensus to Arbitrary Reference Values in Time-Varying Graphs", IEEE Transactions on Automatic Control, vol. 65, no. 4, pp. 1755-1762, April 2020
[7] J. Usevitch and D. Panagou "Determining r- and (r,s)-Robustness of Digraphs Using Mixed Integer Linear Programming", Automatica, vol. 111, 108586, pp. 1-13, January 2020
[8] J. Usevitch and D. Panagou, "Resilient Leader-Follower Consensus with Time-Varying Leaders in Discrete-Time Systems", CDC 2019
[9] K. Garg and D. Panagou, "Control-Lyapunov and Control-Barrier Functions based Quadratic Program for Spatio-temporal Specifications", CDC 2019
[10] K. Garg, E. Arabi and D. Panagou. "Fixed-time control under spatiotemporal and input constraints: A QP based approach," submitted to IEEE TAC, under review, arXiv preprint arXiv:1906.10091 (2019)
[11] K. Garg, E. Arabi and D. Panagou, "Prescribed-time convergence with input constraints: A control Lyapunov function based approach", 2020 American Control Conference, Denver, CO, July 2020
[12] K. Garg and D. Panagou "Finite-Time Stability of Hybrid Systems with Unstable Modes", resubmitted to IEEE TAC, under review.
[13] K. Garg and D. Panagou, "Characterization of Domain of Fixed-time Stability under Control Input Constraints", CDC 2020, under review

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