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NAVAL POSTGRADUATE SCHOOL

Defense against Adversarial Swarms with Parameter Uncertainty

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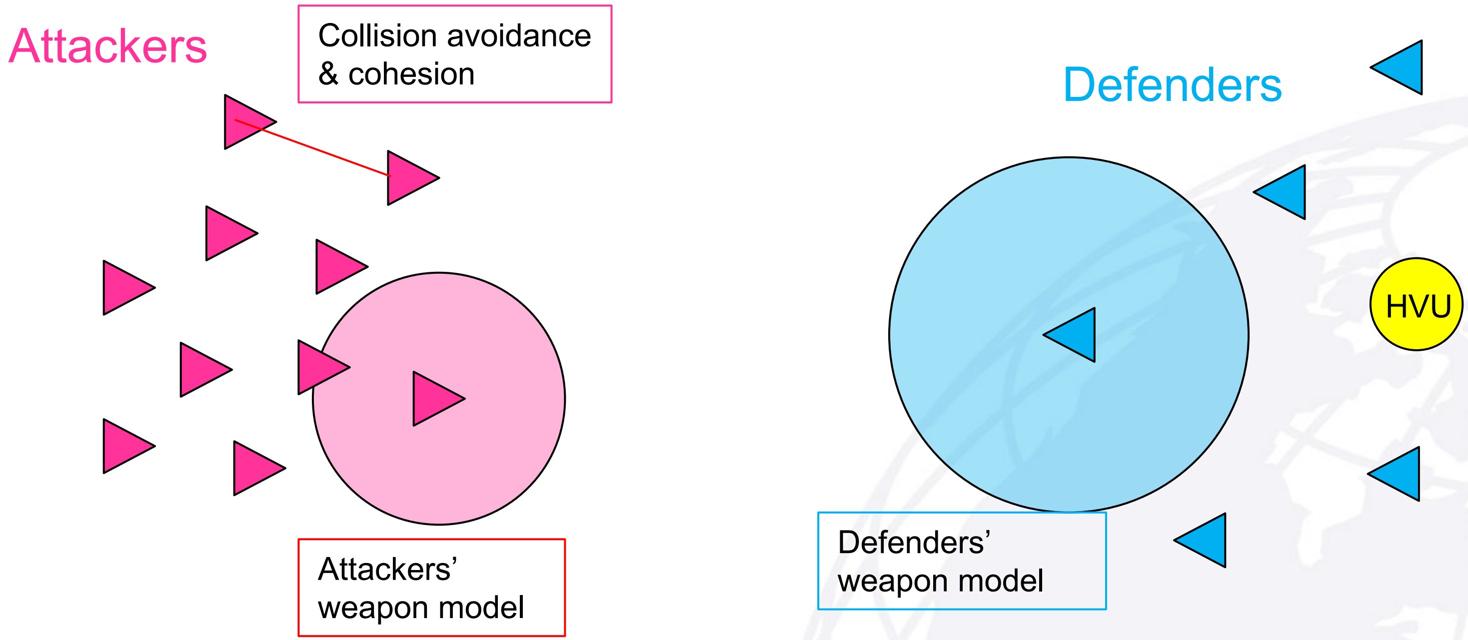
IEEE TCAC Workshop, 2021 CCTA

Multi-Vehicle and Assured Autonomous Control for Aerospace Applications

Introduction



Objectives:

- **Suitable Framework for Modeling Swarm-on-Swarm Engagements**
  - **Performance metrics**
  - **Analysis and synthesis**
  - **Robustness**
- **For a given level of mission success determine**
  - **minimum number of defenders**
  - **optimal defender trajectories**

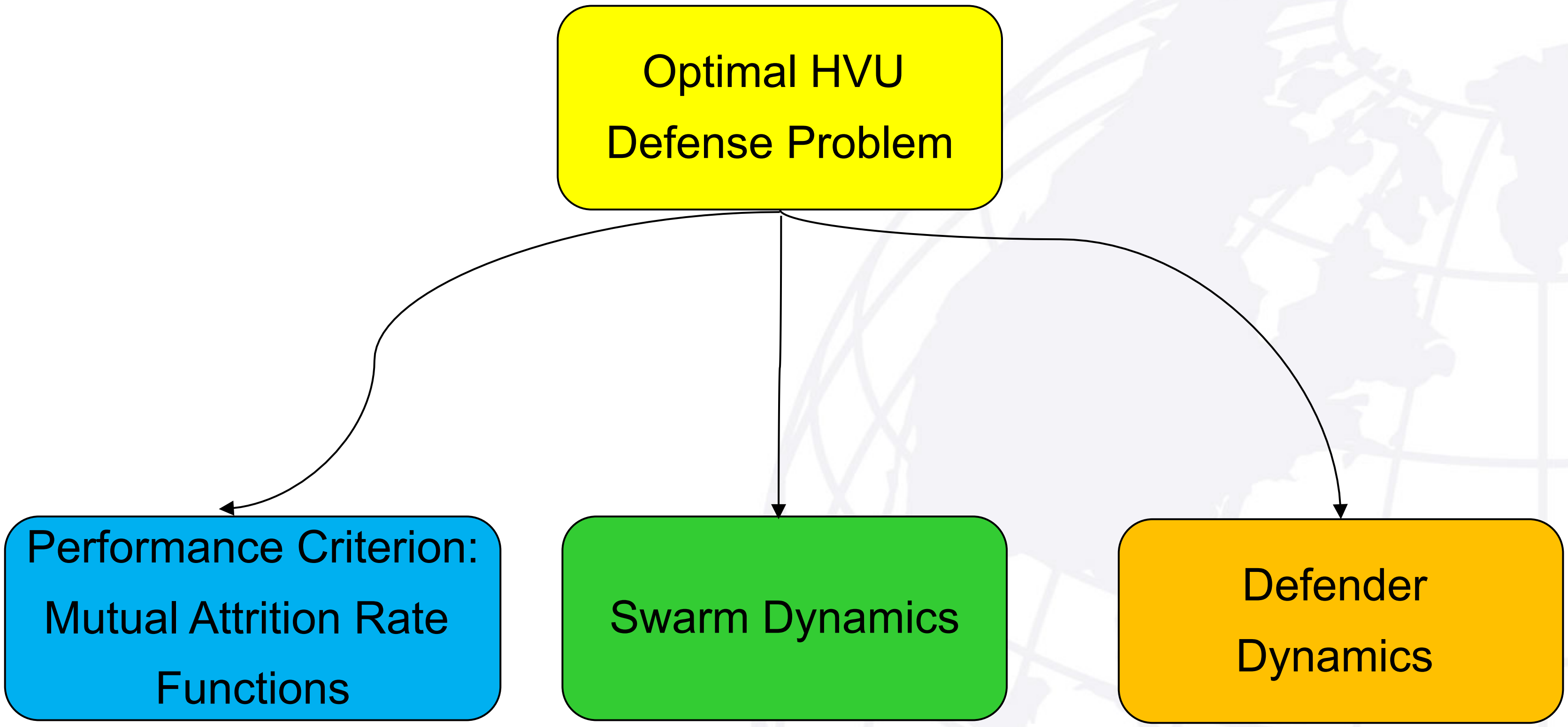
Outline

- Framework: Modeling Swarm on Swarm Engagement as an Optimal Control Problem
- Addressing uncertainty
  - uncertain parameter optimal control
  - estimation
- Trade-offs: black-box robustness
- Conclusions

D. Hambling, "The U.S. Navy Plans To Foil Massive 'Super Swarm' Drone Attacks By Using The Swarm's Intelligence Against Itself," Forbes August 2020

Problem Formulation

- **We seek to maximize probability of HVU survival**

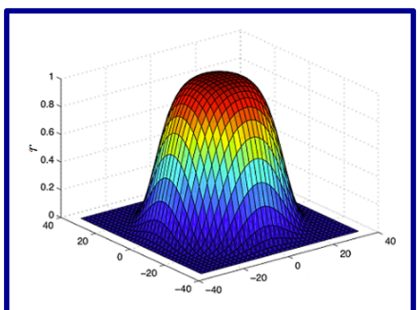


Performance Criterion: Mutual Attrition Modeling

Historical Models:

Sonar/Radar

- instantaneous rate of detection
  - $d(s(t), x(t), t)$
- in time interval $[t, t+\Delta t]$ the probability of detection is given by $d(s(t), x(t), t)\Delta t$



Poisson Scan Model

Let

- $P_{ND}(t)$ – probability of target non-detection at time t,
- $x(t)$ – position of the target at t
- $s(t)$ – position of the searcher at t
- J – probability of target non-detection over a finite time interval $[0, t_f]$

Then

$$P_{ND}(t+\Delta t) = P_{ND}(t)(1 - d(s(t), x(t), t)\Delta t)$$

$$\Rightarrow$$

$$\lim_{\Delta t \to 0} \frac{P_{ND}(t+\Delta t) - P_{ND}(t)}{\Delta t} = -d(s(t), x(t), t)\Delta t)$$

$$\Rightarrow$$

$$\dot{P}_{ND}(t) = -d(s(t), x(t), t)$$

$$\Rightarrow$$

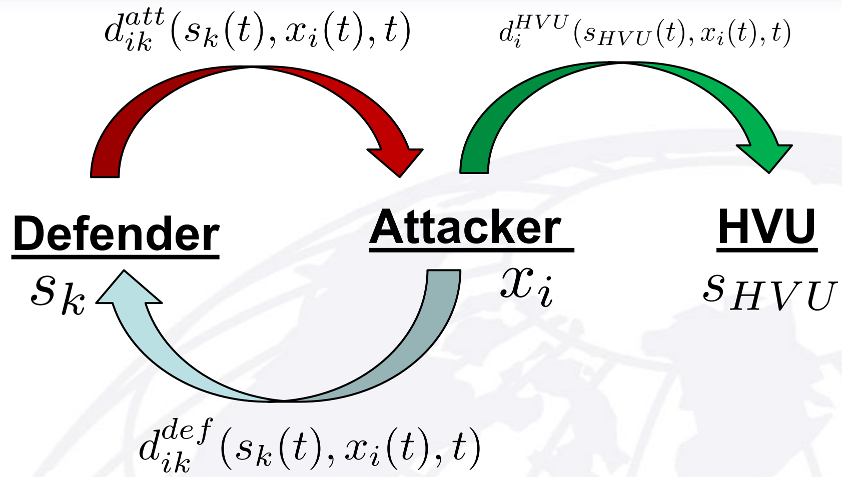
$$P_{ND}(t) = \exp^{-\int_0^t d(s(\tau), x(\tau), \tau) d\tau}$$

$$\Rightarrow$$

$$J = P_{ND}(t_f)$$

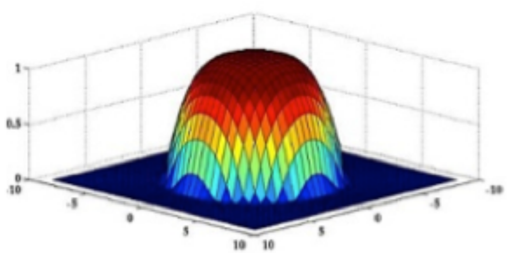
Koopman '46, Stone '77-'80, Washburn 2002

Performance Criterion: Mutual Attrition Modeling

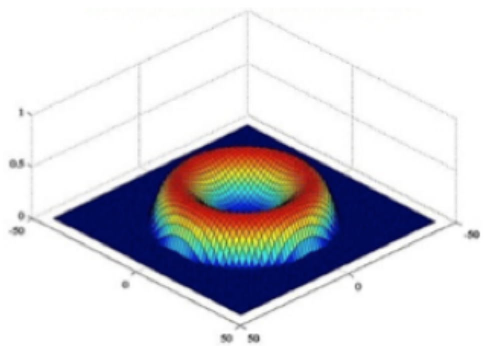


➢ Attrition rates defined by:

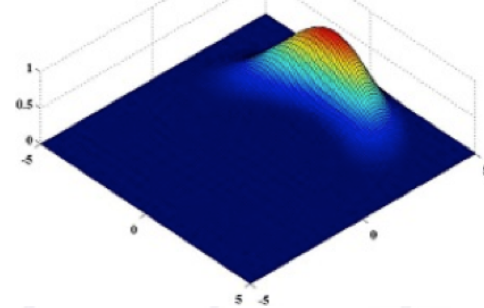
- Distance
- Field-of-View
- Fire Rate



Decreasing firing effectiveness over distance



Maximal firing effectiveness at a distance



Limited by FOV constraints

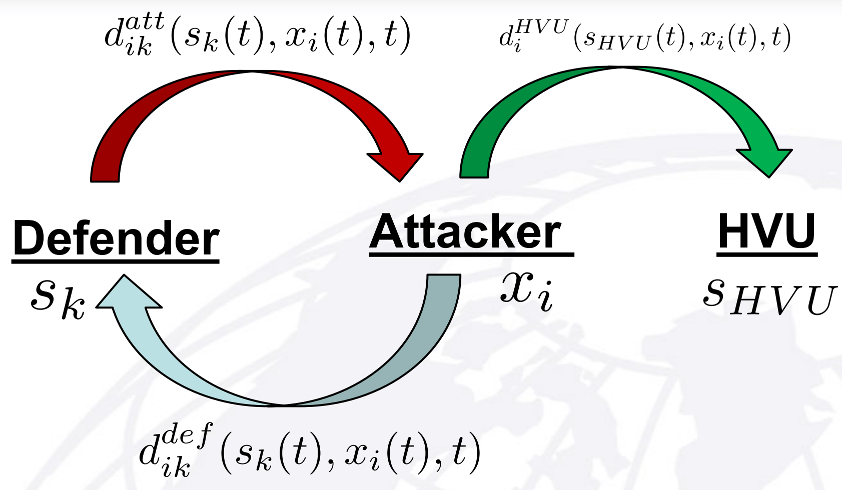
Performance Criterion: Mutual Attrition Modeling

Probabilistic performance metrics:

$Q_i(t)$ = Probability attacker i is alive at time t

$P_k^d(t)$ = Probability defender k is alive at time t

$P(t)$ = Probability HVU is alive at time t



Attacker i must survive attacks from all defenders during Δt

$$Q_i(t+\Delta t) = Q_i(t)\prod_k^M (1 - [d_{ik}^{\text{att}} P_k^d(t)]\Delta t), \implies \dot{Q}_i(t) = -Q_i(t)\sum_k^M (1 - [d_{ik}^{\text{att}} P_k^d(t)]).$$

Same for defenders and HVU

$$P_k^d(t+\Delta t) = P_k^d(t)\prod^N (1 - [d_{ki}^{\text{def}} Q_i(t)]\Delta t), \implies \dot{P}_k^d(t) = -P_k^d(t)\sum_i^N (1 - [d_{ki}^{\text{def}} Q_i(t)]).$$

$$P(t+\Delta t) = P(t)\prod_k^N (1 - [d_k^{\text{hvu}} Q_k(t)]\Delta t). \implies \dot{P}(t) = -P(t)\sum_k^N (1 - [d_k^{\text{hvu}} Q_k(t)])$$

Minimize final cost: HVU non-survival at end of simulation

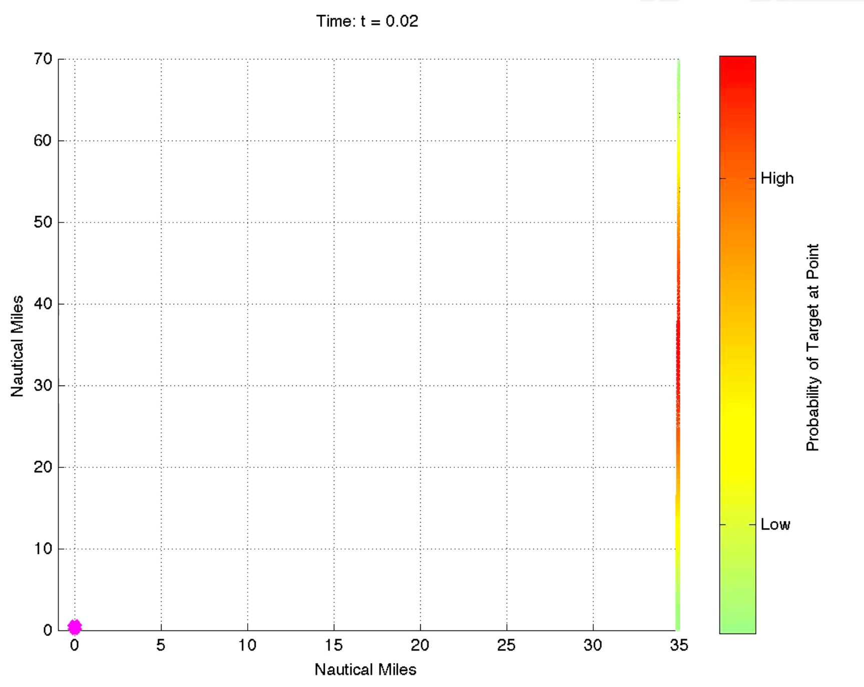
$$J = 1 - P(t_f)$$

Generalization of Lanchester model, Walton et al 2018

Swarm Dynamics

Passive: kamikaze

- **Swarm trajectories are given**
- **Attackers ignore the defenders**



Problem Formulation: Kamikazi case

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$$\min_{u_k}\{J = 1 - P(t_f)\}$$

subject to

N attackers, M defenders

$i = 1, \ldots\ldots N \quad k = 1, \ldots\ldots M$

$$\dot{s}_k = v_k^s$$

$$\dot{v}_k^s = u_k$$

Defender dynamics

$$\dot{Q}_i = -Q_i(t)\sum_k^M (1 - [d_{ik}^{\text{att}} P_k^d(t)])$$

Attacker probability of survival

Attacker trajectories

$$\dot{P}_k^d = -P_k^d(t)\sum_i^N (1 - [d_{ki}^{\text{def}} Q_i(t)])$$

Defender probability of survival

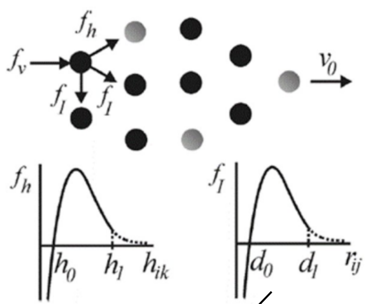
$$\dot{P} = -P(t)\sum_k^N (1 - [d_k^{\text{hvu}} Q_k(t)]).$$

HVU probability of survival

plus constraints on control and collision avoidance

Swarm Dynamics

Active: Decentralized/Potential Based



Intruder evasion

Virtual leader points at HVU

$$\ddot{x}_i = \sum_{j \neq i}^{N} \frac{f_I(x_{ij})}{\|x_{ij}\|} x_{ij} + \boxed{\sum_{k=1}^{M} \frac{f_d(s_{ik})}{\|s_{ik}\|} s_{ik}} + K \frac{h_i}{\|h_i\|} - b\dot{x}_i,$$

P. Ogren, E. Fiorelli, and N. E. Leonard, "Cooperative Control of Mobile Sensor Networks: Adaptive Gradient Climbing in a Distributed Environment," IEEE Trans. Autom. Control, 2004

Problem Formulation Revisited

$$\min_{u_k}\{J = 1 - P(t_f)\}$$

***N* attackers, *M* defenders**

$$i = 1, \ldots.. N \quad k = 1, \ldots.. M$$

subject to

$$\begin{cases} \dot{x}_i = v_i^x \\ \dot{v}_i^x = \sum_{j \neq i}^{N} \frac{f_I(x_{ij})}{\|x_{ij}\|} x_{ij} + \sum_{k=1}^{M} \frac{f_d(s_{ik})}{\|s_{ik}\|} s_{ik} + K \frac{h_i}{\|h_i\|} - b\dot{x}_i \\ \dot{s}_k = v_k^s \\ \dot{v}_k^s = u_k \\ \dot{Q}_i = -Q_i(t) \sum_k^M (1 - [d_{ik}^{\text{att}} P_k^d(t)]) \\ \dot{P}_k^d = -P_k^d(t) \sum_i^N (1 - [d_{ki}^{\text{def}} Q_i(t)]) \\ \dot{P} = -P(t) \sum_k^N (1 - [d_k^{\text{hvu}} Q_k(t)]). \end{cases}$$

Attacker dynamics (first two equations)

Defender dynamics (next two equations)

Attacker probability of survival

Defender probability of survival

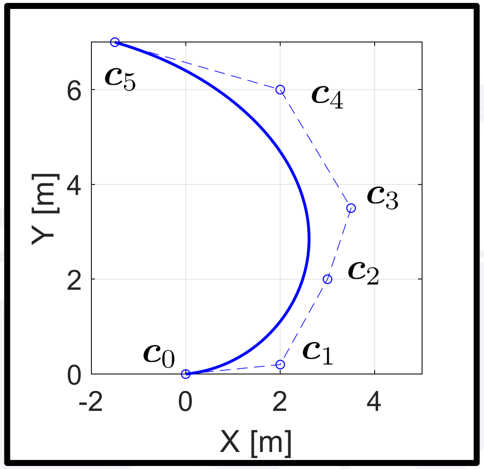
HVU probability of survival

plus constraints on control and collision avoidance

Bernstein Polynomials

A degree *n* Bernstein polynomial is given by

$$\boldsymbol{x}_N(t) = \sum_{k=0}^{N} \boldsymbol{c}_k b_{k,N}(t)$$



where

- $b_{k,N}(t)$ are the Bernstein polynomial basis

$$b_{k,N} = \binom{N}{k} t^N (t_f - t)^{N-k}, \quad t \in [0, t_f]$$

- $\mathbf{c}_k \in \mathbb{R}^3$ are the *Bernstein coefficients*

Sergei Bernstein (1880-1968)

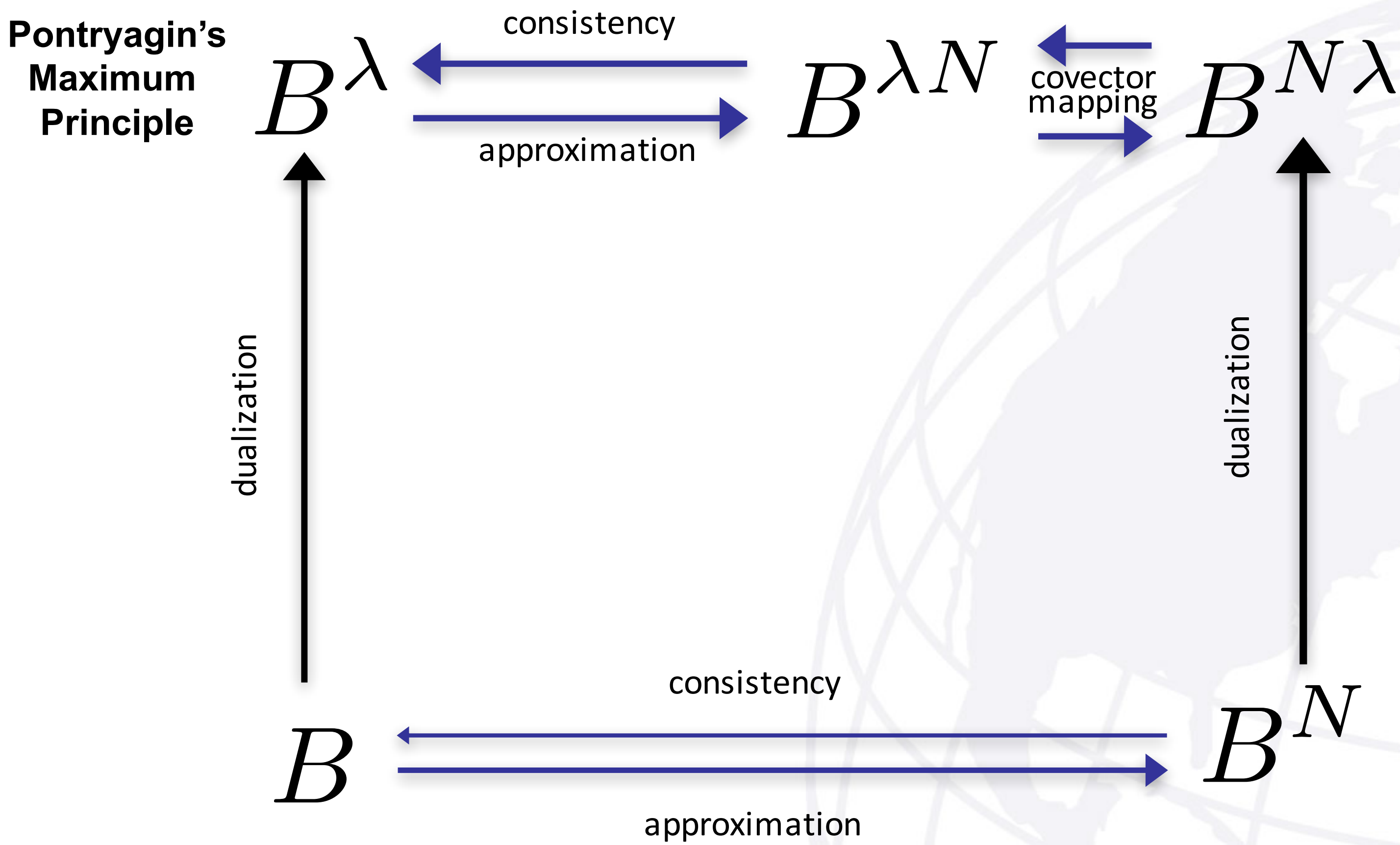
Paul de Casteljau (1930)

Pierre Bézier (1910-1999)

R. Farouki, “The Bernstein Polynomial Basis: A Centennial Retrospective,” Computer Aided Geometric Design, 2012

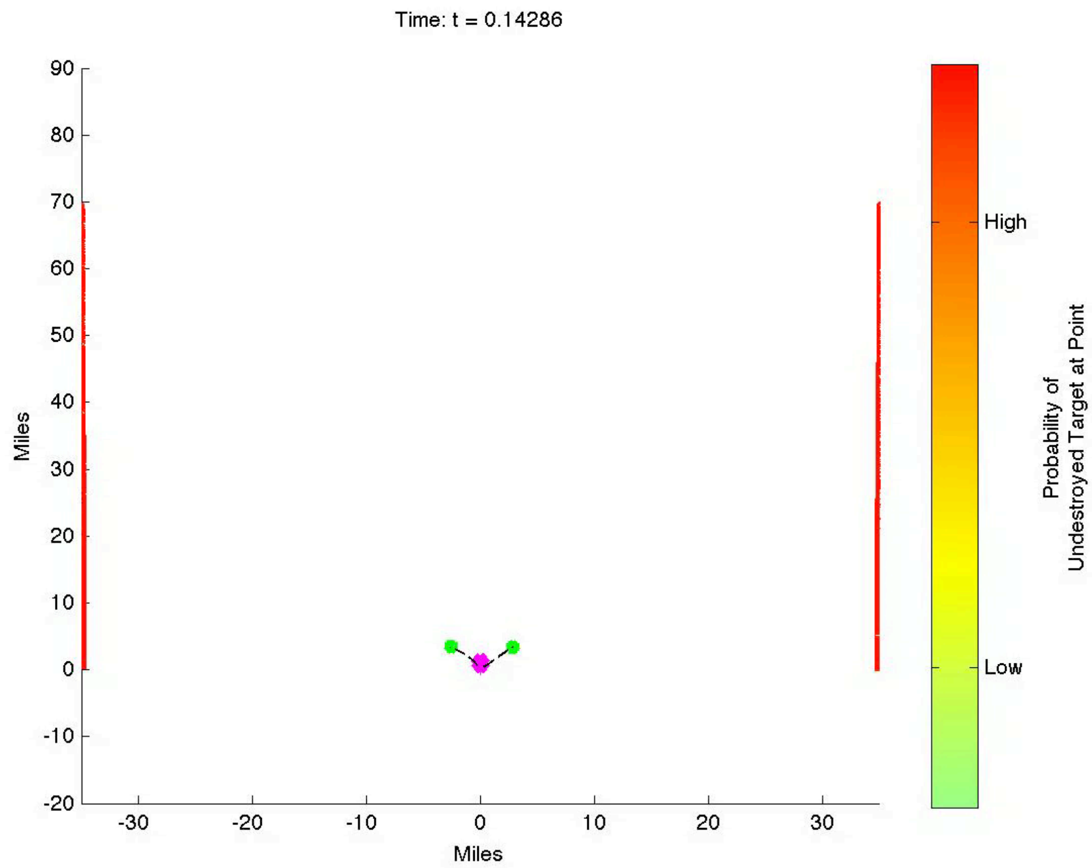
Theoretical Issues

All the way down to KKT multipliers…



Ross et al 2008 – 2011, Cichella et al 2020, 2021

Results: Kamikazi Swarm

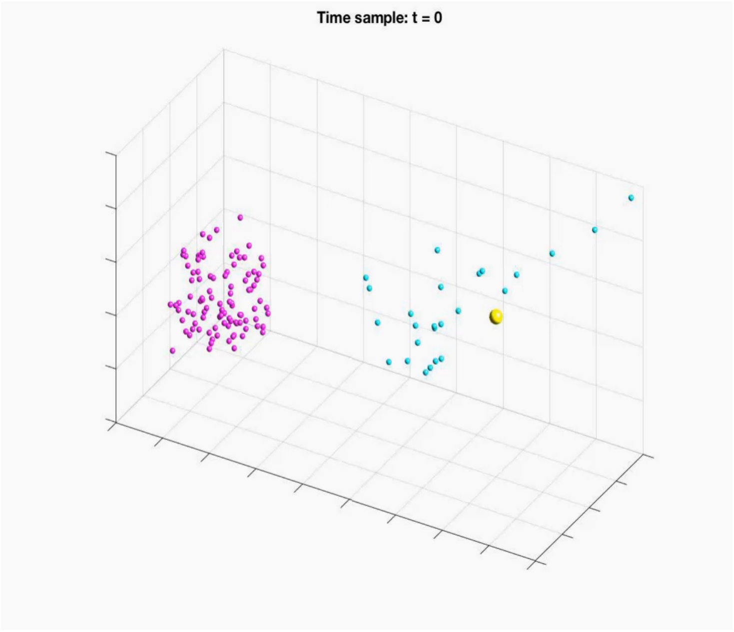


Results: Leonard Swarm

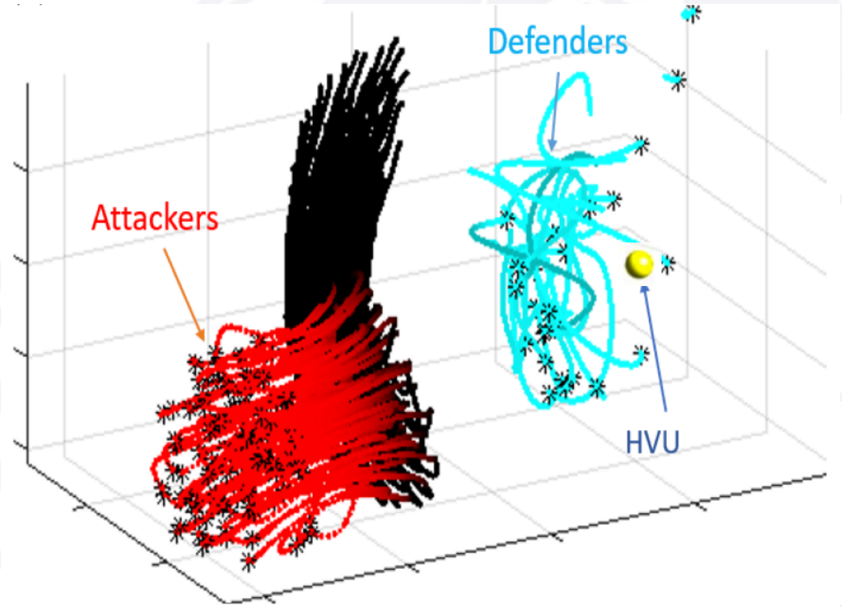
Does optimization help?

100 attackers versus 25 defenders with double weapons range and fire rate

Unoptimized defender trajectories



Optimized defender trajectories



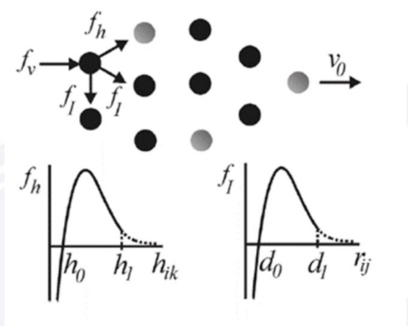
Backup Slides

Parameter Uncertainty

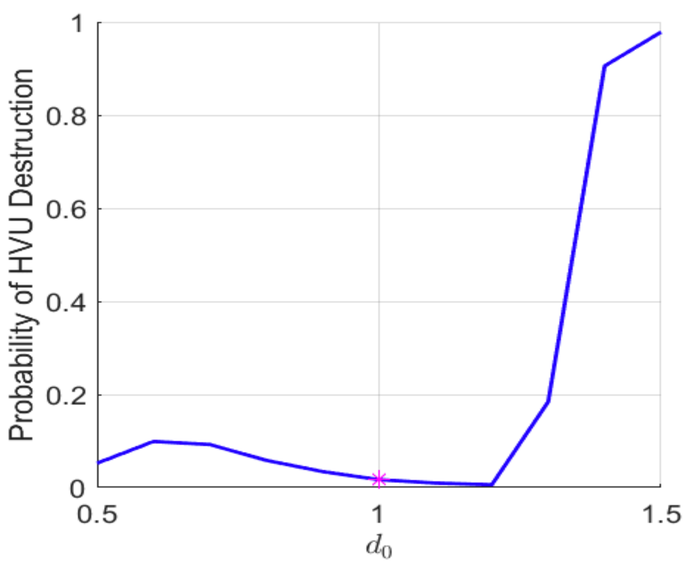
What about uncertainty?

Recall the Leonard swarm dynamics

$$\ddot{x}_i = \sum_{j \neq i}^{N} \frac{f_I(x_{ij})}{\|x_{ij}\|} x_{ij} + \sum_{k=1}^{M} \frac{f_d(s_{ik})}{\|s_{ik}\|} s_{ik} + K \frac{h_i}{\|h_i\|} - b\dot{x}_i,$$



Suppose d_0 is uncertain in the range [0.5, 1.5]



Problem Formulation must explicitly account for uncertainty in d_0

$$J = \int_{\omega \in \Omega} (1 - P(t_f, \omega))\phi(\omega) d\omega$$
$$\omega = d_0$$
$$\Omega = [0.5, 1.5]$$
$$\phi(\omega) = 1$$

Problem Formulation

Approach: optimize over all parameter values

1. Characterize parameter space → $\omega \in \Omega, \ \phi(\omega)$

1. Track state dynamics over all possible values → $x(0,\omega) = x_0(\omega)$, $\dot{x}(t,\omega) = f(x(t,\omega), u(t), \omega)$

1. Optimize cost over entire performance profile → $\min_u J$

$$J = \int_\Omega \left(F(x(T,\omega),\omega) + \int_0^T r(x(t,\omega), u(t), t, \omega) dt \right) \phi(\omega) d\omega$$

Integrate over multi-dimensional parameter space; calculate metrics such as expectation or variance

Control Inputs → *Spectrum of Possible Systems* → Expected Performance

Problem Formulation

Uncertain Parameter Optimal Control Framework:

Problem B: Given $\phi : \Omega \to \mathbb{R}$, determine the control $u : [0, T] \to U \subset \mathbb{R}^{n_u}$ that minimizes the cost functional:

$$J = \int_{\Omega} \left(F(x(T,\omega),\omega) + \int_0^T r(x(t,\omega),u(t),t,\omega)dt \right) \phi(\omega) d\omega$$

subject to:

$$\begin{aligned} \dot{x}(t,\omega) &= f(x(t,\omega),u(t),\omega) \\ x(0,\omega) &= x_0(\omega) \\ g(u(t)) &\leq 0 \end{aligned}$$

- New Maximum Principleof Optimal Control, Gabasov and Kirilova, 1974
- Ensemble Control, Brockett 1997, ….
- Application of polynomial chaos in stability and control, Hover and Triantafyllou, 2006, ……
- Unscented Control, Ross, Karpenko and Proulx 2016,…
- Maximum Principle for Deep Learning, Li, Chen, Tai, E, 2018, ….

- Efficient numerical algorithms needed.

Numerical Approach

Step 1: *discretize parameter space*

B

$$\begin{cases} J = \int_{\Omega} \left(F(x(T,\omega),\omega) + \int_0^T r(x(t,\omega),u(t),t,\omega)dt \right) \phi(\omega)d\omega \\ \dot{x}(t,\omega) = f(x(t,\omega),u(t),\omega) \\ x(0,\omega) = x_0(\omega) \end{cases}$$

Assumption: For each $M \in \mathbb{N}$, there is a set of nodes $\{\omega_i^M\}_{i=1}^M \subset \omega$ and an associated set of weights $\{\alpha_i^M\}_{i=1}^M \subset \mathbb{R}$, such that for any continuous function $h : \omega \to \mathbb{R}$,

$$\int_{\omega} h(\omega)d\omega = \lim_{M\to\infty} \sum_{i=1}^M h(\omega_i^M)\alpha_i^M.$$

B^M

$$\begin{cases} J^M = \sum_{i=1}^M \left(F(x_i^M(T,\omega_i^M),\omega_i^M) + \int_0^T r(x_i^M(t),u(t),t,\omega_i^M)dt \right) \phi(\omega_i^M)\alpha_i^M d\omega \\ \dot{x}_i^M(t,\omega_i^M) = f(x_i^M(t,\omega_i^M),u(t),\omega_i^M)) \qquad i = 1,\ldots,M \\ x_i^M(0,\omega_i^M) = x_0(\omega_i^M) \\ g(u(t)) \le 0 \text{ for all } t \in [0,T] \end{cases}$$

Numerical Approach

B

$$\begin{cases} J = \int_{\Omega} \left(F(x(T,\omega),\omega) + \int_0^T r(x(t,\omega),u(t),t,\omega)dt \right) \phi(\omega)d\omega \\ \dot{x}(t,\omega) = f(x(t,\omega),u(t),\omega) \\ x(0,\omega) = x_0(\omega) \end{cases}$$

B^{M}

$$\begin{cases} J^M = \sum_{i=1}^{M} \left(F(x_i^M(T,\omega_i^M),\omega_i^M) + \int_0^T r(x_i^M(t),u(t),t,\omega_i^M)dt \right) \phi(\omega_i^M)\alpha_i^M d\omega \\ \dot{x}_i^M(t,\omega_i^M) = f(x_i^M(t,\omega_i^M),u(t),\omega_i^M)) \qquad i = 1,\ldots,M \\ x_i^M(0,\omega_i^M) = x_0(\omega_i^M) \\ g(u(t)) \leq 0 \text{ for all } t \in [0,T] \end{cases}$$

- **Step 2: *solve approximate problem***
  - Problem B^{M} is a standard Mayer Bolza optimal control problem

Numerical Approach

$$\mathbf{B}\begin{cases} J = \int_\Omega \left(F(x(T,\omega),\omega) + \int_0^T r(x(t,\omega),u(t),t,\omega)dt \right)\phi(\omega)d\omega \\ \dot{x}(t,\omega) = f(x(t,\omega),u(t),\omega) \\ x(0,\omega) = x_0(\omega) \end{cases}$$

$$\mathbf{B}^{\mathbf{M}}\begin{cases} J^M = \sum_{i=1}^{M} \left(F(x_i^M(T,\omega_i^M),\omega_i^M) + \int_0^T r(x_i^M(t),u(t),t,\omega_i^M)dt \right)\phi(\omega_i^M)\alpha_i^M d\omega \\ \dot{x}_i^M(t,\omega_i^M) = f(x_i^M(t,\omega_i^M),u(t),\omega_i^M)) \qquad i = 1,\ldots,M \\ x_i^M(0,\omega_i^M) = x_0(\omega_i^M) \\ g(u(t)) \leq 0 \text{ for all } t \in [0,T] \end{cases}$$

- **Discretizing $\mathbf{B}^{\mathbf{M}}$ yields very sparse NLP**

$$\mathbf{B}^{\mathbf{MN}}\begin{cases} J^{MN} = \sum_{i=1}^{M} \left(F(\bar{x}_{Mi}^{NN},\omega_i^M) + \sum_{k=0}^{N} r(\bar{x}_M^{Nk},\bar{u}^{Nk},t_k,\omega_i^M)b_k^N \right)\phi(\omega_i^M)a_i^M \\ D^N\bar{x}_M^N - f(\bar{x}_M^N,\bar{u}^N,\bar{\omega}^M) = 0, \qquad i = 1,\ldots,M \\ \bar{x}_M^{N0} = \bar{x}_0 \\ g(\bar{u}^{Nk}) \leq 0 \text{ for all } k = 0,\ldots,N \end{cases}$$

Theoretical Issues

What do we need to prove?

$$\text{Problem } B \quad \underset{\text{approximation}}{\overset{?}{\rightleftarrows}} \quad \text{Problem } B^M$$

- **Feasibility**
  - Solutions created by approximate problem are actually feasible for original
- **Consistency**
  - If optimal solutions to the approximate problem converge, they converge to optimal of original

Theoretical Issues

Feasibility & Consistency

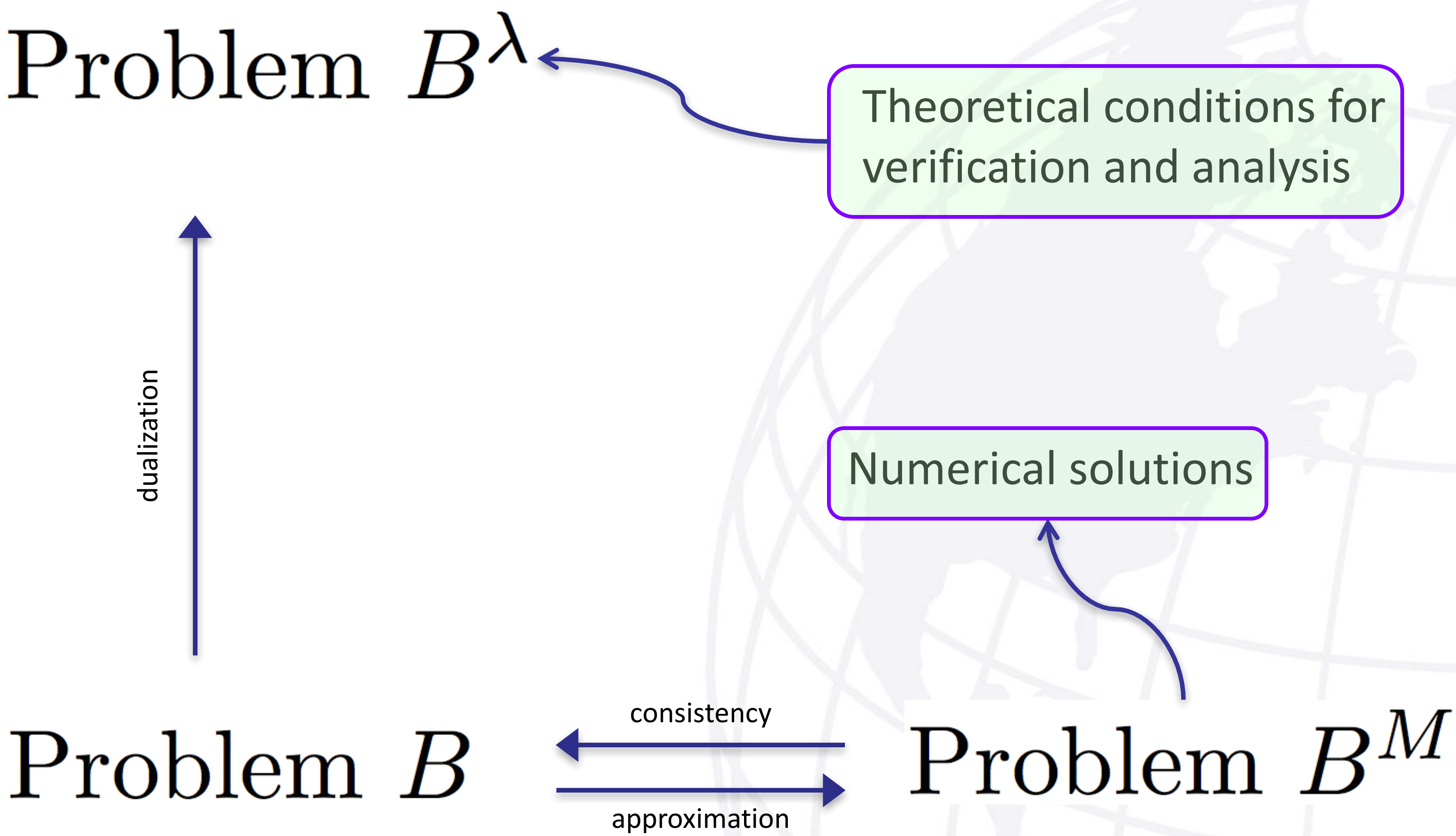
Theorem: Let $\{u_M^*\}_{M \in V}$ be a sequence of optimal controls for Problem B^M with an accumulation point u^∞. Then u^∞ is an optimal control for Problem B.

Definition 1. **Uniform Accumulation Point -** A function f is called a uniform accumulation point of the sequence of functions $\{f_n\}_{n=0}^\infty$ if $\exists$ a subsequence of $\{f_n\}_{n=0}^\infty$ that uniformly converges to f. Similarly, a vector $v \in \mathbb{R}^M$ is called a uniform accumulation point of the sequence of vectors $\{v_n\}_{n=0}^\infty$ if $\exists$ a subsequence of $\{v_n\}_{n=0}^\infty$ that converges to v.

- **Convergent subsequences of optimal controls**

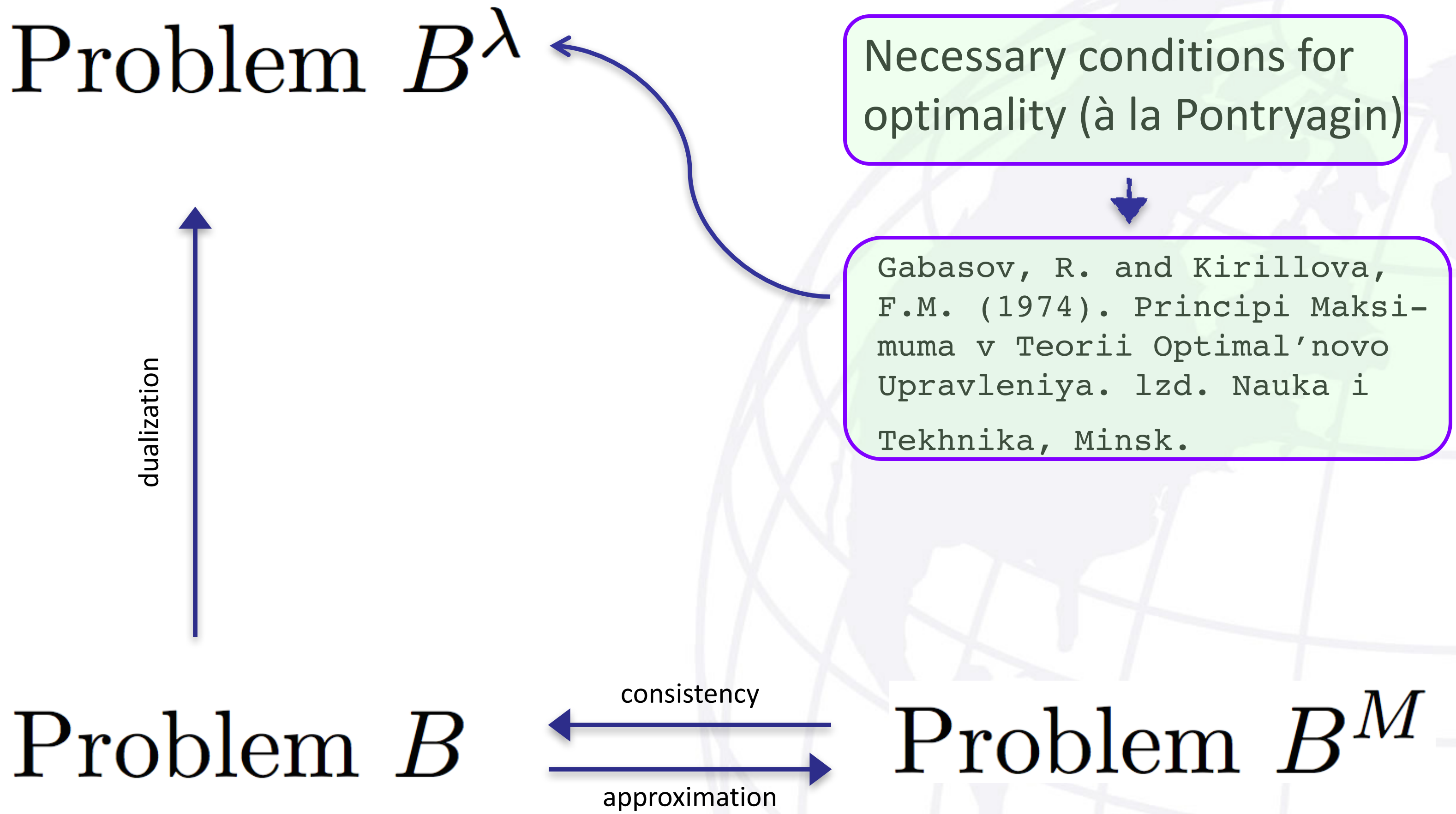
Theoretical Issues

A more general problem solving structure



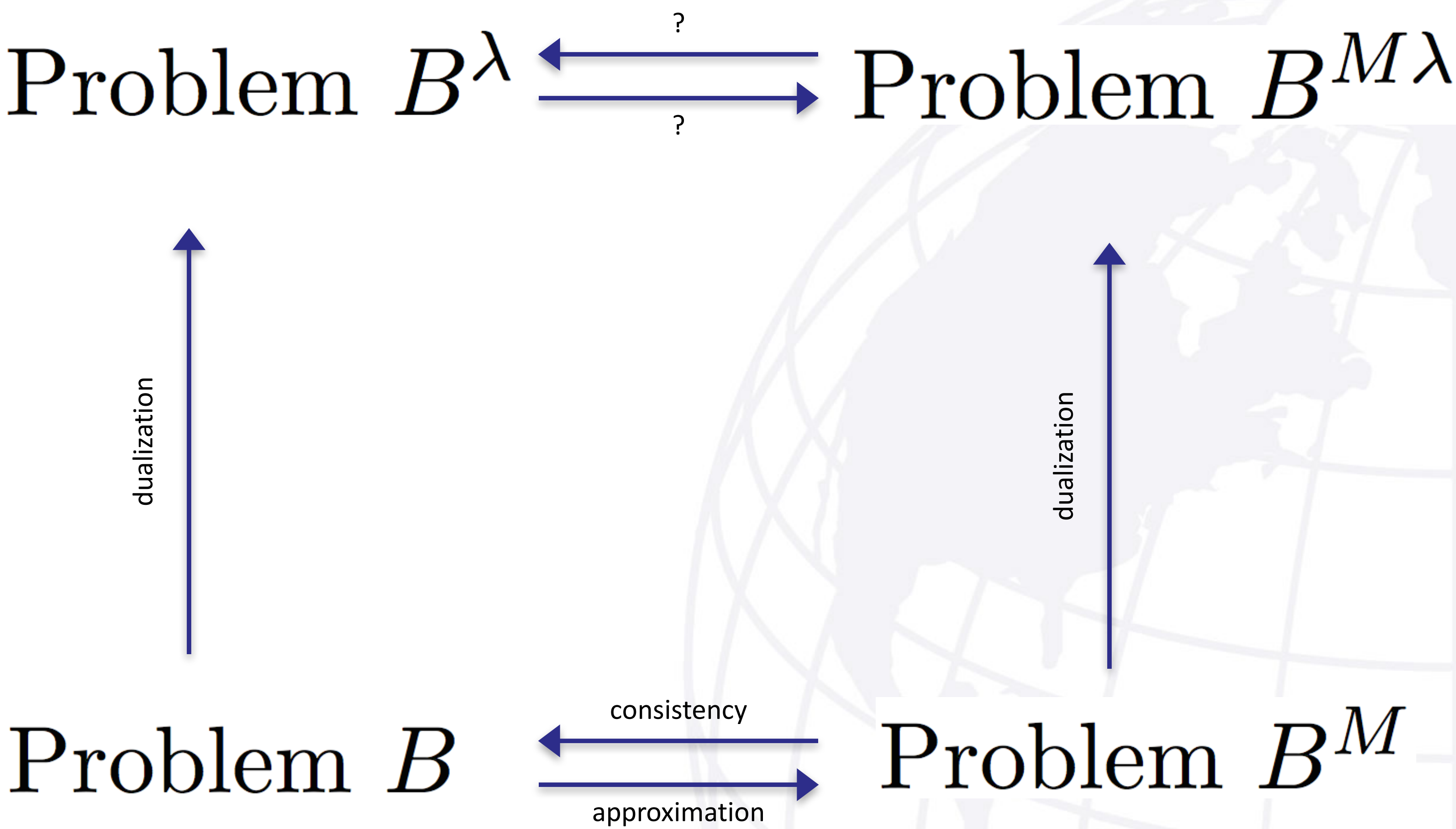
Theoretical Issues

A more general problem solving structure



Theoretical Issues

Are the dual problems consistent?



Theoretical Issues

$$\frac{\partial \lambda(t,\omega)}{\partial t} = -\frac{\partial \tilde{H}}{\partial x} \qquad \lambda(T,\omega) = \left.\frac{\partial F}{\partial x}\right|_{\Omega}$$

Gabasov, R. and Kirillova, F.M. (1974). Principi Maksimuma v Teorii Optimal'novo Upravleniya. lzd. Nauka i Tekhnika, Minsk.

$$\tilde{H}(x,\lambda,u,t,\omega) = \lambda^T f(x,u,\omega) + r(x,u,t,\omega)$$

$$\mathbf{H}(x,\lambda,u,t) = \int_{\Omega} \tilde{H}(x,\lambda,u,t,\omega)\,d\omega$$

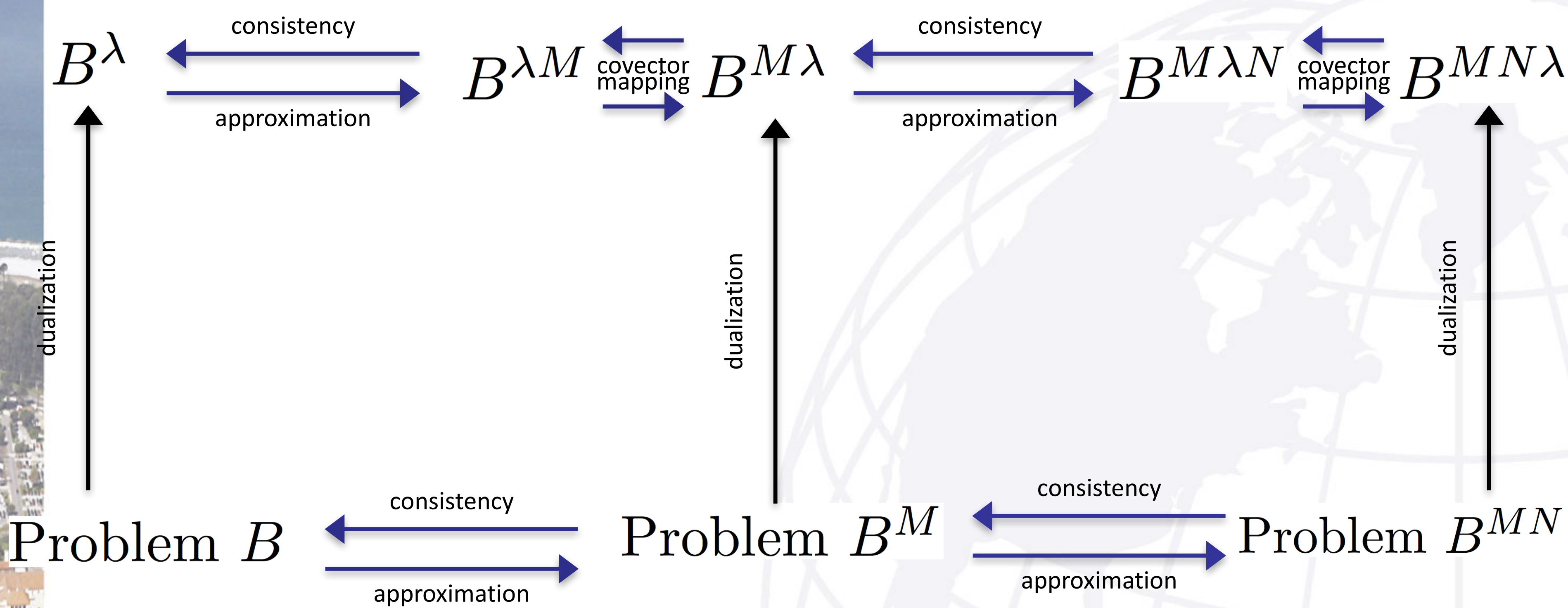
- **Hamiltonian Minimization Principle through consistency**

Theorem: Let $\{u^*_M\}$ be a sequence of optimal controls for Problem $\mathbf{B}^{\mathbf{M}}$ with an accumulation point u^∞. Let $(x^\infty, \lambda^\infty)$ be the primal and dual variables for Problem $\mathbf{B}$ created by the control u^∞. Then for all feasible u:

$$\mathbf{H}(x^\infty, \lambda^\infty, u^\infty, t) \le \mathbf{H}(x^\infty, \lambda^\infty, u, t)$$

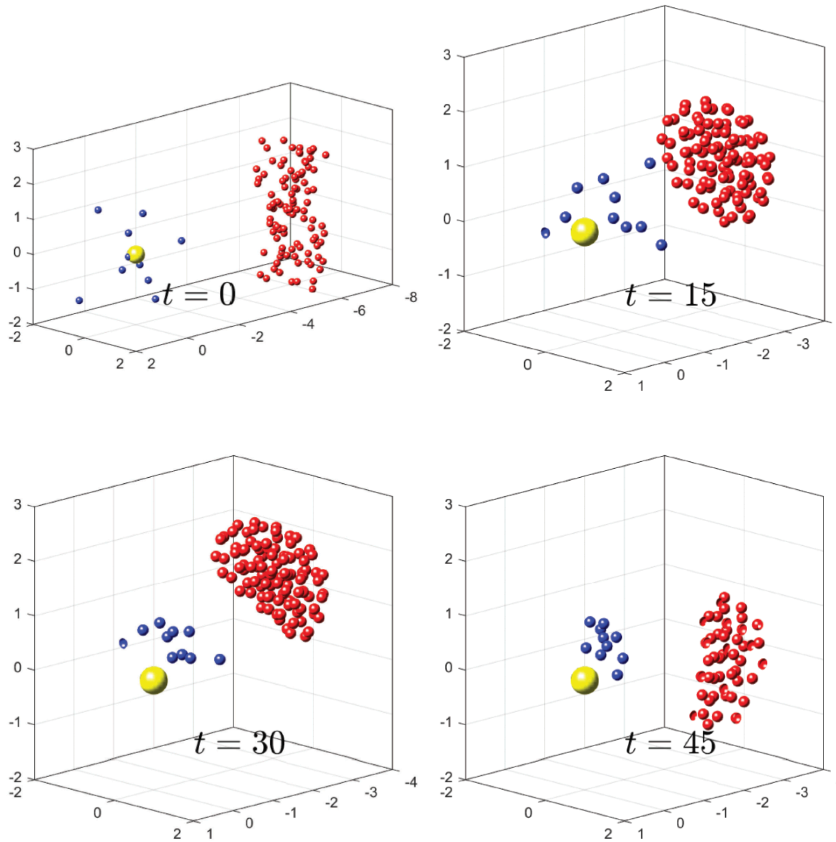
Theoretical Issues

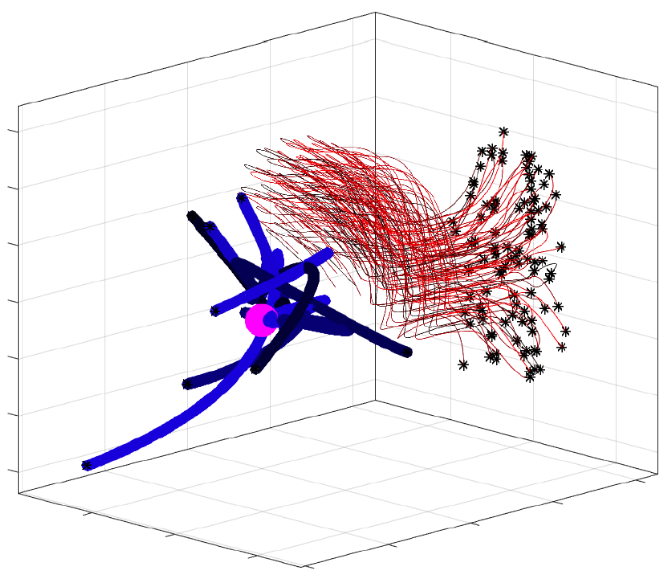
All the way down to KKT multipliers…



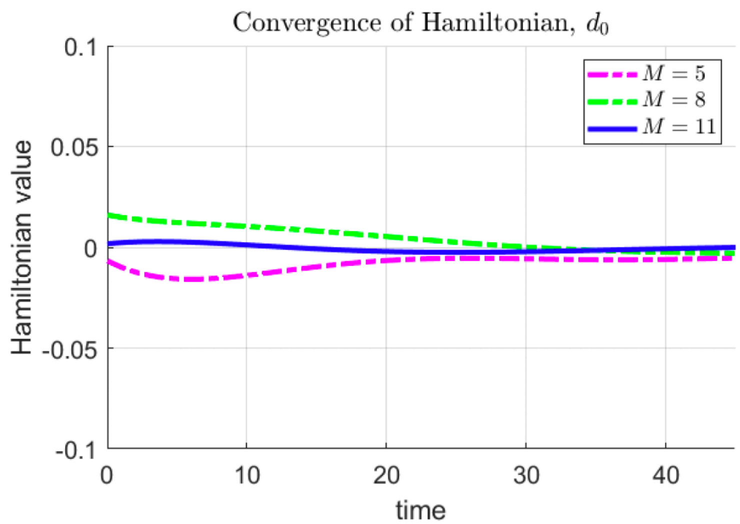
Phelps et al 2014, Walton et al 2019, 2021, Ross et al 2016,

Example: Swarm Engagement

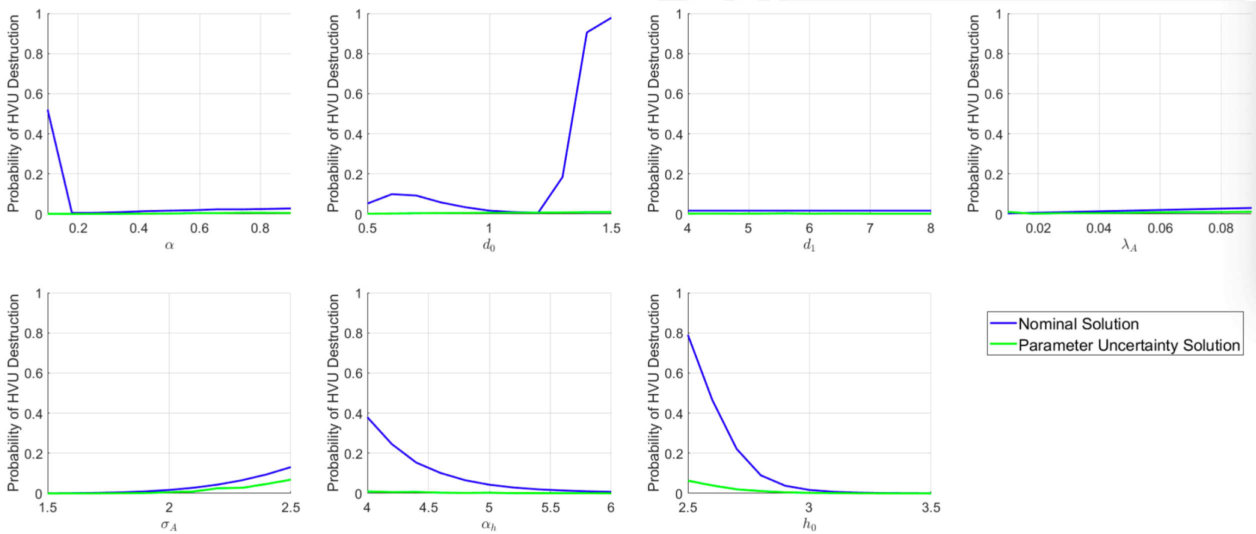




Example: Swarm Engagement



Robustness

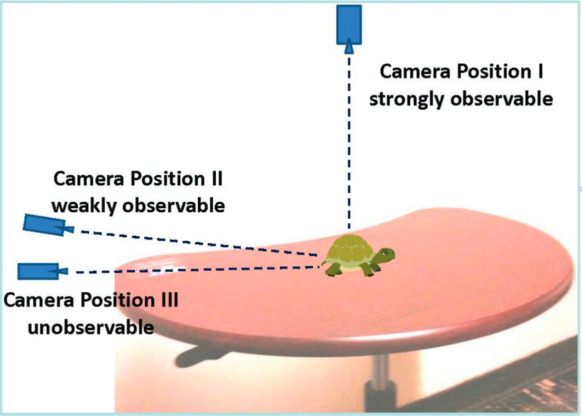


Estimation of Swarm Parameters

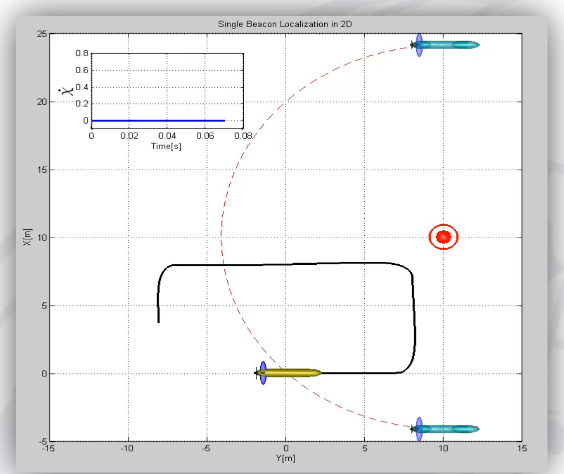
Challenges

Nonlinear Observability

Sensor locations matter



Trajectories and Observation windows matter



➢ **Challenges**

- **Non-cooperative swarm**
  - **unknown control inputs**
- **Optimal sensor/observer placement**
- **Big Data - partial observability**
- **Small observation window**

Krener 1977, Kang 2012, Pascoal 2014

Observability of Linear Systems

- Let
$$\dot{x} = Ax$$
$$y = Cx$$
- Then the system is observable iff the observability Gramian
$$G = \int_0^T e^{A^T\tau} C^T C e^{A\tau} d\tau > 0, \forall T > 0$$
- Consider
$$\varepsilon = \min_{\delta x(0), t \in [0,T]} \left\| y(t, \hat{x}(t)) - y(t, x(t)) \right\|$$
subject to
$$\dot{\hat{x}} = A\hat{x}, \quad \hat{x}(0) = x(0) + \delta x(0), \quad \delta x(0) \in R^n$$
$$\left\| \delta x(0) \right\| = \rho \quad \Leftarrow \quad \text{estimation ambiguity}$$
- Solution $\varepsilon = \left(\sqrt{\lambda_{\min}(G)}\right)\rho$ or $\dfrac{\rho}{\varepsilon} = \dfrac{1}{\sqrt{\lambda_{\min}(G)}}$
- Unobservability index ρ/ε small – good, large - bad

Kang et al 2009, 2017

Partial Observability of Linear Systems

- Let $\dot{x} = Ax$

$$y = Cx$$

$$z = Px, \text{ e.g. } z = [x_1, \ldots\ldots, x_{n_z}]^T, \; n_z \leq n_x$$

- Consider

$$\rho^2 = \max_{x \in R^n} \{\|Px\|^2\}$$

subject to

$$x^T G x \leq \epsilon^2$$

- Define

$$L = x^T G x - \lambda (x^T G x - \epsilon^2)$$

- Then

$$\lambda^* = \frac{\rho^2}{\varepsilon^2}$$

Optimal Lagrange Multiplier = Square of Unobservability index of *z*

Partial Observability of Non-Linear Systems

- Consider

$$\dot{x} = f(t, x(t), u(t), \mu)$$ - system dynamics

$$y = h(t, x(t), u(t), \mu)$$ - measured output

$$z = Px(t)$$ - desired estimates

- Definition: Unobservability Index

Given a trajectory $(x(t), \mu),\ t \in [t_0, t_1]$ and $\rho > 0$.

The unobservability index of $(x(0), \mu)$ is the ratio ρ/ε, where

$$\rho = \max_{(\hat{x}(0), \hat{\mu})} \left\| \hat{z} - z \right\|$$

subject to

$$\left\| h(t, \hat{x}(t), \hat{u}(t), \hat{\mu}) - h(t, x(t), u(t), \mu) \right\| \le \varepsilon,$$

$$\dot{\hat{x}} = f(t, \hat{x}(t), \hat{u}(t), \hat{\mu})$$

Empirical Observability Gramian

- Let the inner product of y

$$\langle y, y \rangle = y^T y$$

Let $\{w_1, w_2, \cdots, w_{n_x}\}$ be a basis of W and $v_0 = (x_0, \mu_0)$ Define

$$\Delta_i = \frac{1}{2\rho} \int_{t_0}^{t_1} \left(y(t, v_0 + \rho w_i) - y(t, v_0 - \rho w_i) \right) dt$$

$$G_Y = \left(\langle \Delta_i, \Delta_j \rangle \right)_{i,j=1}^{n_z}$$

Then for small perturbations ρ, unobservability index

$$\rho / \varepsilon \approx \sqrt{\frac{1}{\lambda_{\min}(G_Y)}}$$

Moore 1981, Marsden 2002, Singh 2005,2006, Krener 2009, Kang 2009-2014, Serpas 2012, Morgensen 2015

Partial Observability of Non-Linear Systems

- Consider

$$\dot{x} = f(t, x(t), u(t), \mu) \quad \text{- system dynamics}$$
$$y = h(t, x(t), u(t), \mu) \quad \text{- measured output} \qquad (1)$$
$$z = Px \quad \text{- partial state}$$

- Let G_Y be the empirical observability Gramian of (1)
- Consider

$$\rho^2 = \max_{x \in R^n} \{\|Px\|^2\}$$
$$\text{subject to}$$
$$x^T G_Y x \leq \epsilon^2$$
$$x_{i_{min}} \leq x_i \leq x_{i_{max}}$$

- The bounds on x_i represent user knowledge
- Then $\lambda^* = \dfrac{\rho^2}{\varepsilon^2}$

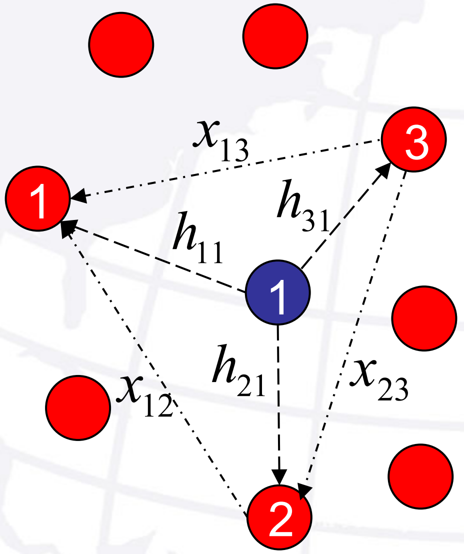
Example: Attacking Swarm

- Swarm model
  - Distributed autonomous control framework
  - Using virtual leaders and artificial potential functions

- Example scenario
  - One virtual leader and 5 followers
  - Point mass in plane with fully actuated dynamics

$$\ddot{x}_i = u_i, \quad i = 1 \cdots 5$$

Leonard et al 2001, 2004



Example: Attacking Swarm

- Control law

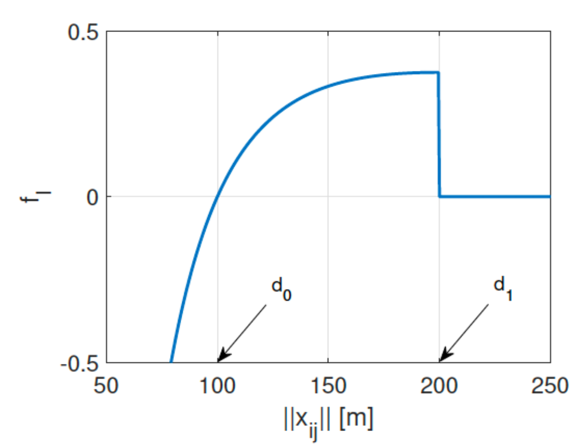
$$u_i = -\sum_{j\neq i}^{5} \frac{f_I(x_{ij})}{\|x_{ij}\|} x_{ij} - \sum_{k=1}^{1} \frac{f_h(h_{ik})}{\|h_{ik}\|} h_{ik} - K\dot{x}_i$$

- Unknown parameters α_I, d_0, d_1 in interaction force magnitude f_I, the gain K and initial position and velocity of the virtual leader

$$f_I = \begin{cases} \nabla_{\|x_{ij}\|} V_I, & 0 < \|x_{ij}\| < d_1 \\ 0, & \|x_{ij}\| \geq d_1 \end{cases}$$

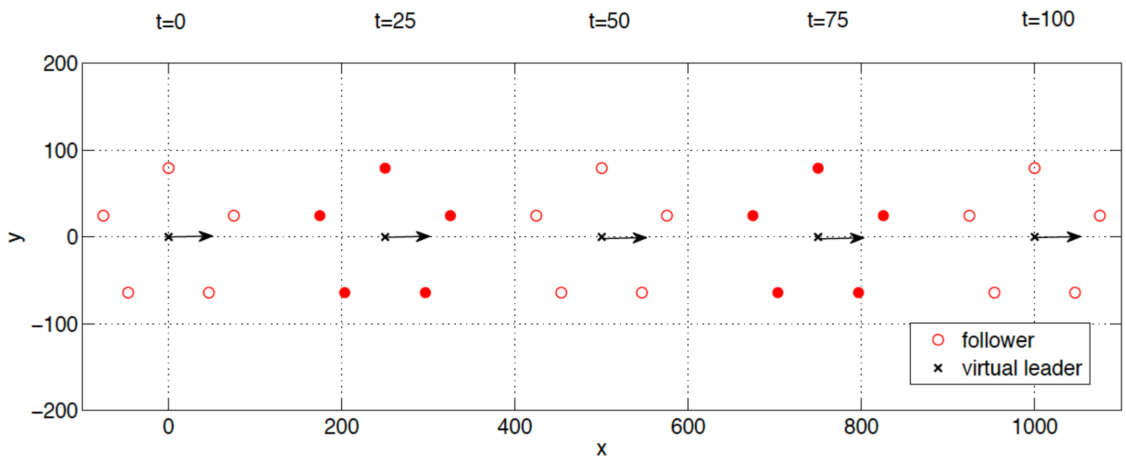
where

$$V_I = \begin{cases} \alpha_I \left(\ln\left(\|x_{ij}\|\right) + \frac{d_0}{\|x_{ij}\|} \right), & 0 < \|x_{ij}\| < d_1 \\ 0, & \|x_{ij}\| \geq d_1 \end{cases}$$



Example: Attacking Swarm

- Scenario 1: Swarm in steady state

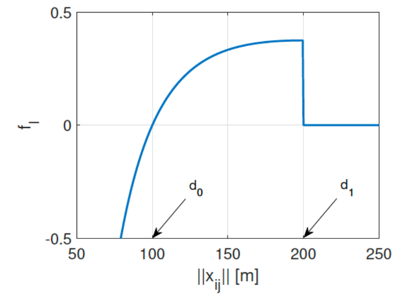


- Unobservability index $\rho/\varepsilon = \infty$ **Unobservable!!**
- However, partial unobservability index is small for

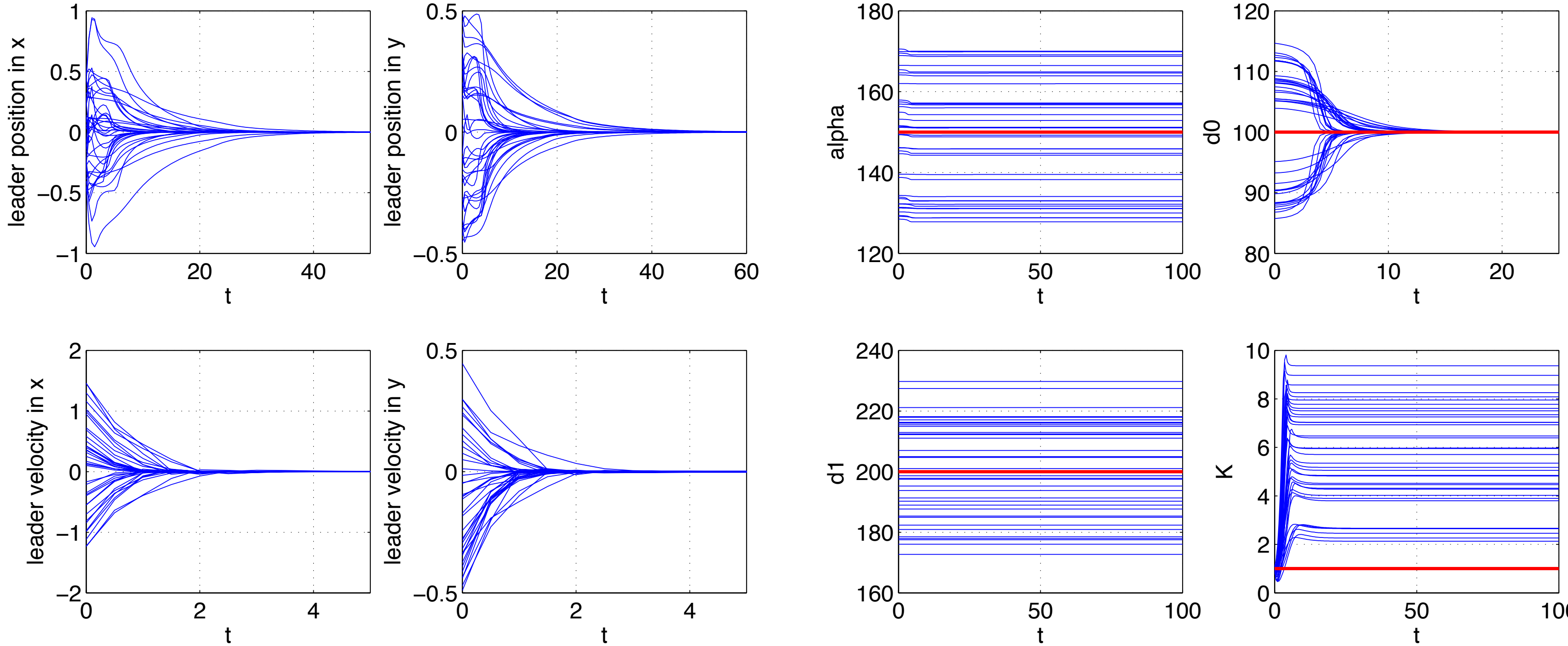
Estimation Variable (z)	Unobservability Index (ρ/ϵ)
Leader Position	1.779×10^{-1}
Leader Velocity	1.698×10^{-2}
Parameter α	$9.640 \times 10^{+3}$
Parameter d_0	6.208×10^{-3}
Parameter d_1	$2.000 \times 10^{+4}$
Parameter K	$1.000 \times 10^{+2}$

Example: Attacking Swarm

- **Scenario 1: Swarm in steady state**



- **UKF correctly estimates partially observable states**

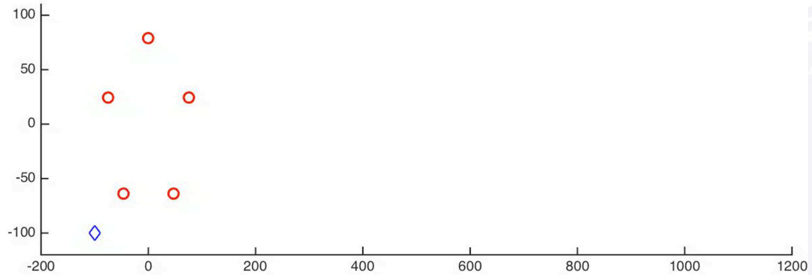


estimation errors of the virtual leader

true parameters UKF estimations

Example: Attacking Swarm

Scenario 2: disrupt using an intruder, 100 sec observation window



	With an intruder
Unobservability index ρ/ε	**1.424**

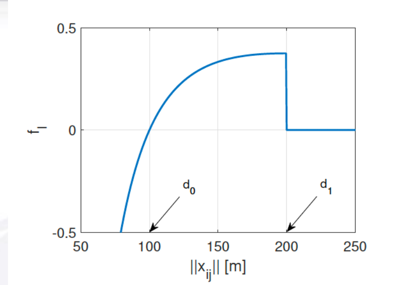
Observable!!

Partial Observability Analysis

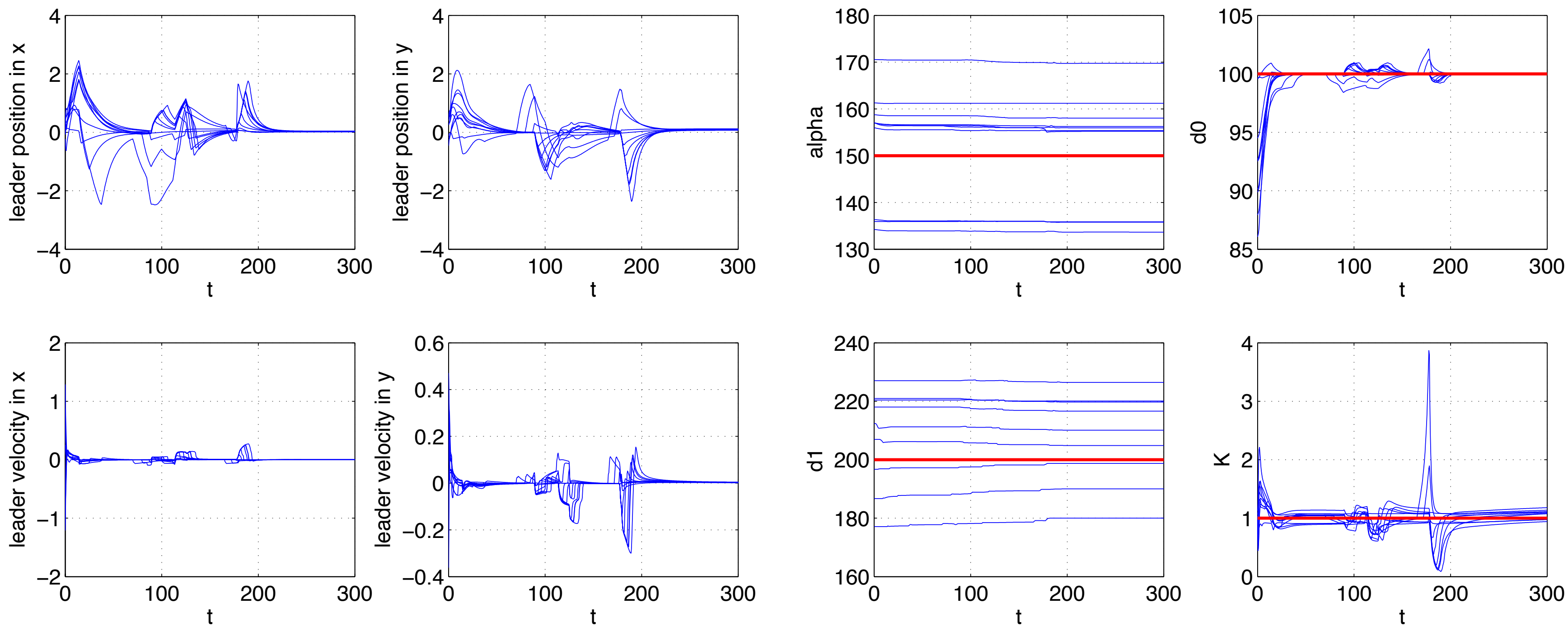
Estimation Variable (z)	Unobservability Index (ρ/ϵ)
Leader Position	2.231×10^{-1}
Leader Velocity	2.355×10^{-2}
Parameter α	1.958×10^{-1}
Parameter d_0	5.628×10^{-3}
Parameter d_1	1.099×10^{-2}
Parameter K	1.927×10^{-1}

Example: Attacking Swarm

Estimation of Parameters



- UKF results:

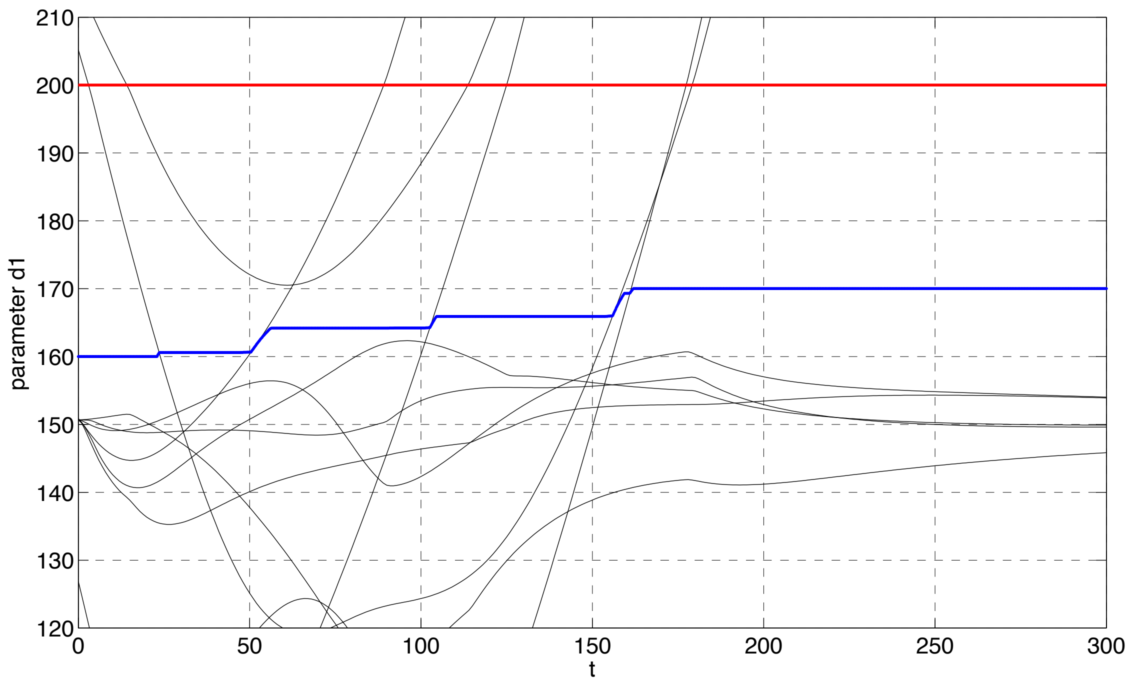


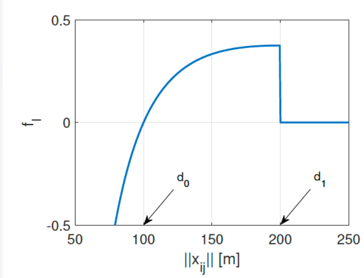
estimation errors of the virtual leader

true parameters — UKF estimations

Estimation of Parameters

- UKF results: estimate only d_1 assuming all others are known





− true value

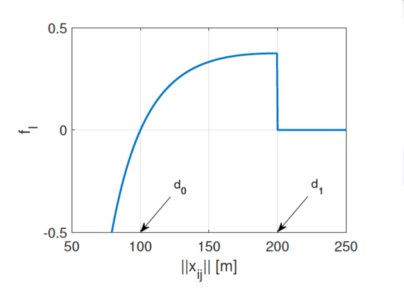
− UKF estimation

− relative distances among agents

d_1 defines discontinuity in agent dynamics and is observable on a set of measure zero

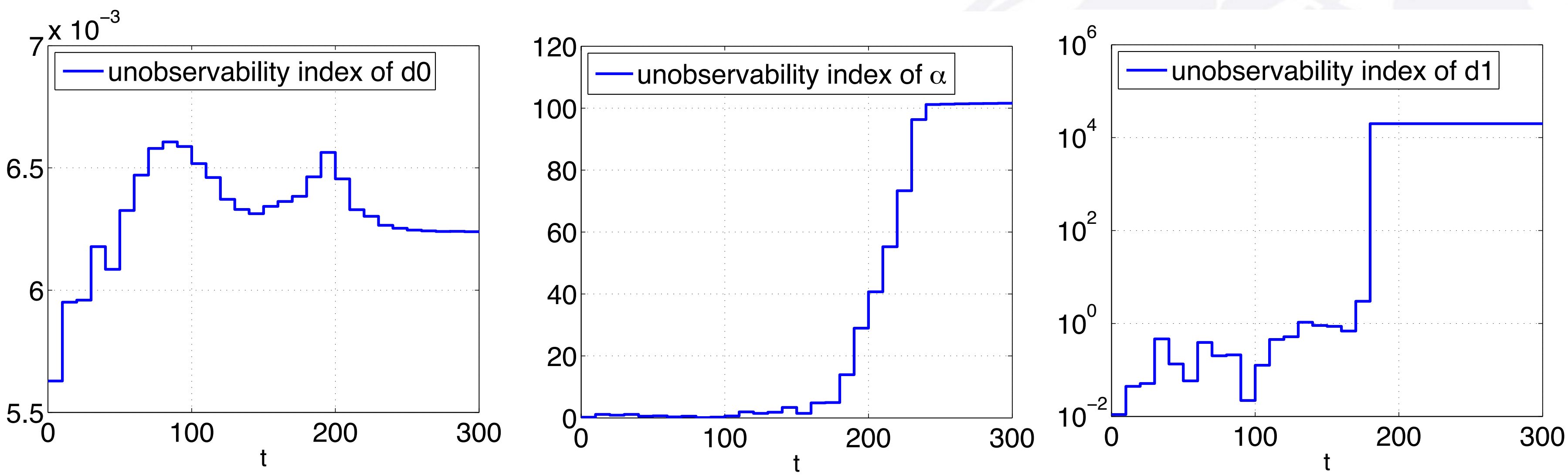
Example: Attacking Swarm

Estimation of Parameters



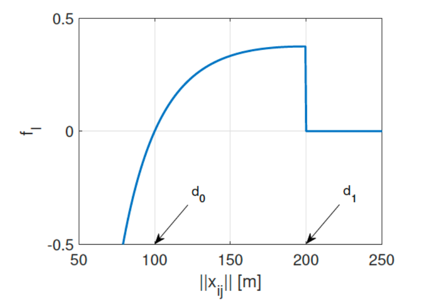
- UKF results (from the time intruder enters the swarm):

observation window matters!



observation window: [t, t+100]

Estimation of Parameters



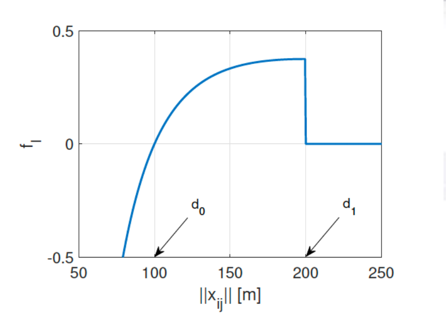
- Optimization using partial observability analysis
  - Step 1: estimate i.c. of the virtual leader and d_0

$$
\begin{cases}
\text{Find} & \hat{x}_l(0) \in R^2, \;\; \hat{\dot{x}}_l(0) \in R^2, \text{ and } \hat{p} = [\hat{\alpha}, \hat{d}_0, \hat{d}_1, \hat{K}] \in \mathbb{R}^4 \text{ to} \\
\text{minimize} & J(\hat{x}_l(0), \hat{\dot{x}}_l(0), \hat{p}) = \log\left(1 + \int_0^{100} \|\hat{y}(t) - y(t)\|^2_{W_y} dt\right) \\
\text{subject to} & \dot{\hat{x}} = f(\hat{x}, \hat{p}, t) \\
 & \hat{x}(0) = [\hat{x}_l^T(0), \hat{\dot{x}}_l^T(0), x_1^T(0), \dot{x}_1^T(0), \cdots, x_5^T(0), \dot{x}_5^T(0)]^T \\
 & \hat{y}(t) = [\hat{x}_1^T(t), \hat{\dot{x}}_1^T(t), \cdots, \hat{x}_5^T(t), \hat{\dot{x}}_5^T(t)]^T
\end{cases}
$$

estimation variable (z)	estimation error
$x_l(0) = (0,0)$	2.395×10^{-4}
$\dot{x}_l(0) = (10,0)$	2.991×10^{-6}
$\alpha = 150$	21.76
$d_0 = 100$	2.967×10^{-5}
$d_1 = 200$	19.104
$K = 1$	7.190

- The reported estimation error is averaged over 10 runs from random initial guesses +/- 50% of true value.
- Optimizer: SNOPT
- Average runtime is 247 s (MacBook Pro 2.3GHz i7 with 8 GB memory)

Estimation of Parameters



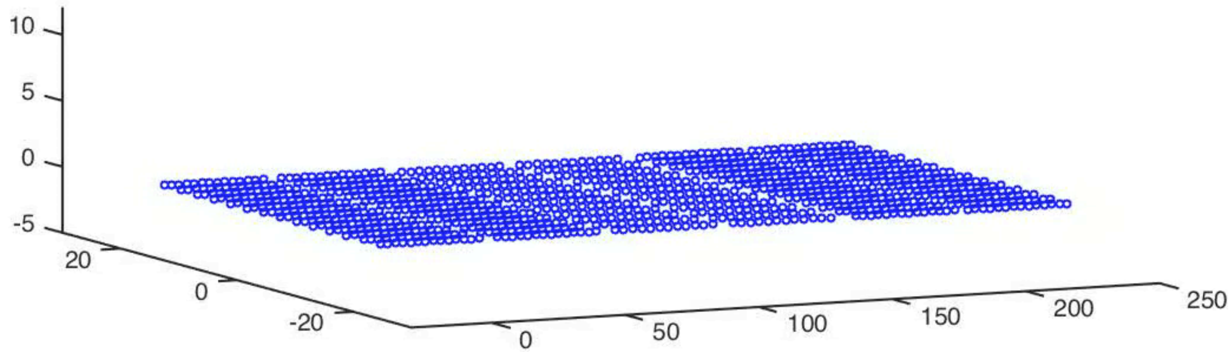
- Optimization using partial observability analysis
  - Step 2: use estimates in Step 1 to obtain the rest

$$
\begin{cases}
\text{Find} & \hat{\alpha},\ \hat{d}_1,\ \text{and}\ \hat{K} \quad \text{to} \\
\text{minimize} & J(\hat{\alpha}, \hat{d}_1, \hat{K}) = \log\left(1 + \int_0^{100} \|\hat{y}(t) - y(t)\|^2_{W_y} dt\right) \\
\text{subject to} & \dot{\hat{x}} = f(\hat{x}, \hat{p}, t) \\
 & \hat{x}(0) = [\hat{x}_l^T(0), \dot{\hat{x}}_l^T(0), x_1^T(0), \dot{x}_1^T(0), \cdots, x_5^T(0), \dot{x}_5^T(0)]^T \\
 & \hat{y}(t) = [\hat{x}_1^T(t), \dot{\hat{x}}_1^T(t), \cdots, \hat{x}_5^T(t), \dot{\hat{x}}_5^T(t)]^T
\end{cases}
$$

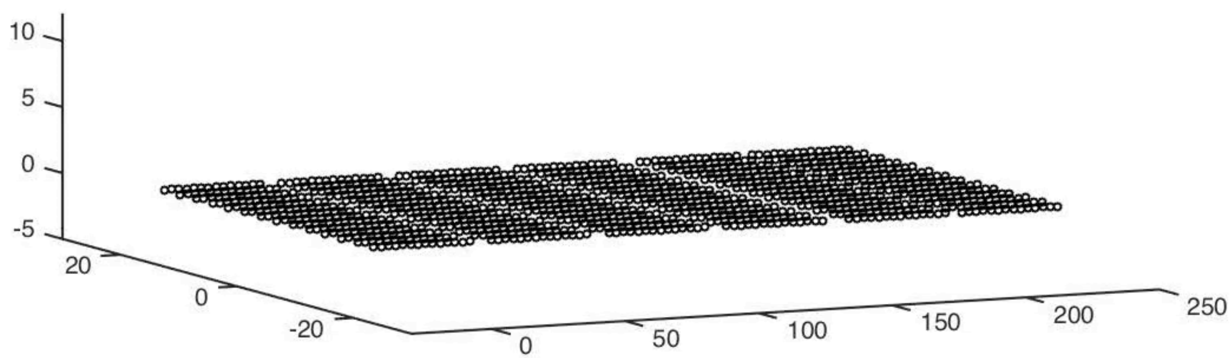
estimation variable (z)	estimation error
$\alpha = 150$	1.117×10^{-2}
$d_1 = 200$	2.915×10^{-4}
$K = 1$	6.610×10^{-5}

- The reported estimation error is averaged over 10 runs from random initial guesses +/- 50% of true value.
- Optimizer: SNOPT
- Average runtime is 398 s (MacBook Pro 2.3GHz i7 with 8 GB memory)

Towards Large Scale Swarm Models

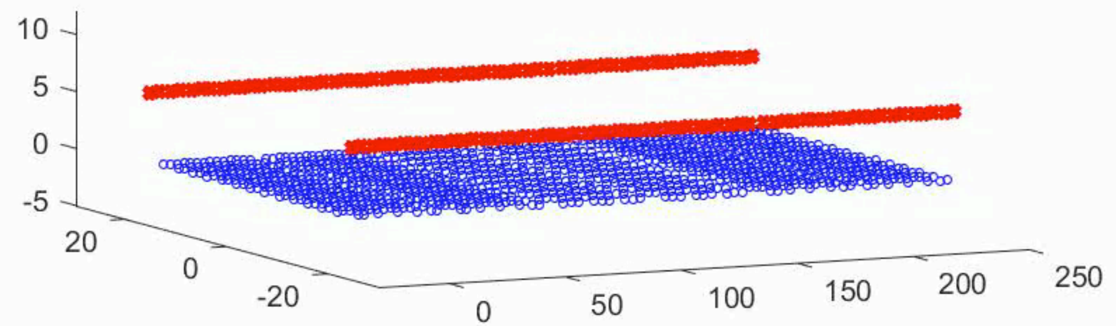


Swarm A
1200 agents

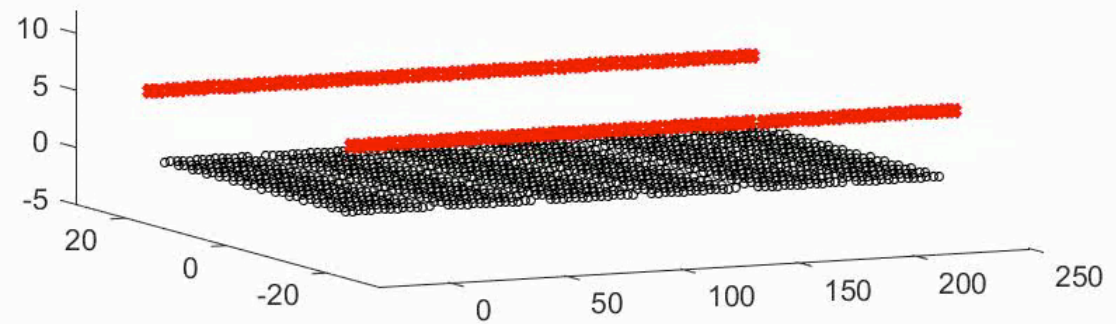


Swarm B
1200 agents

Towards Large Scale Swarm Models



Swarm A

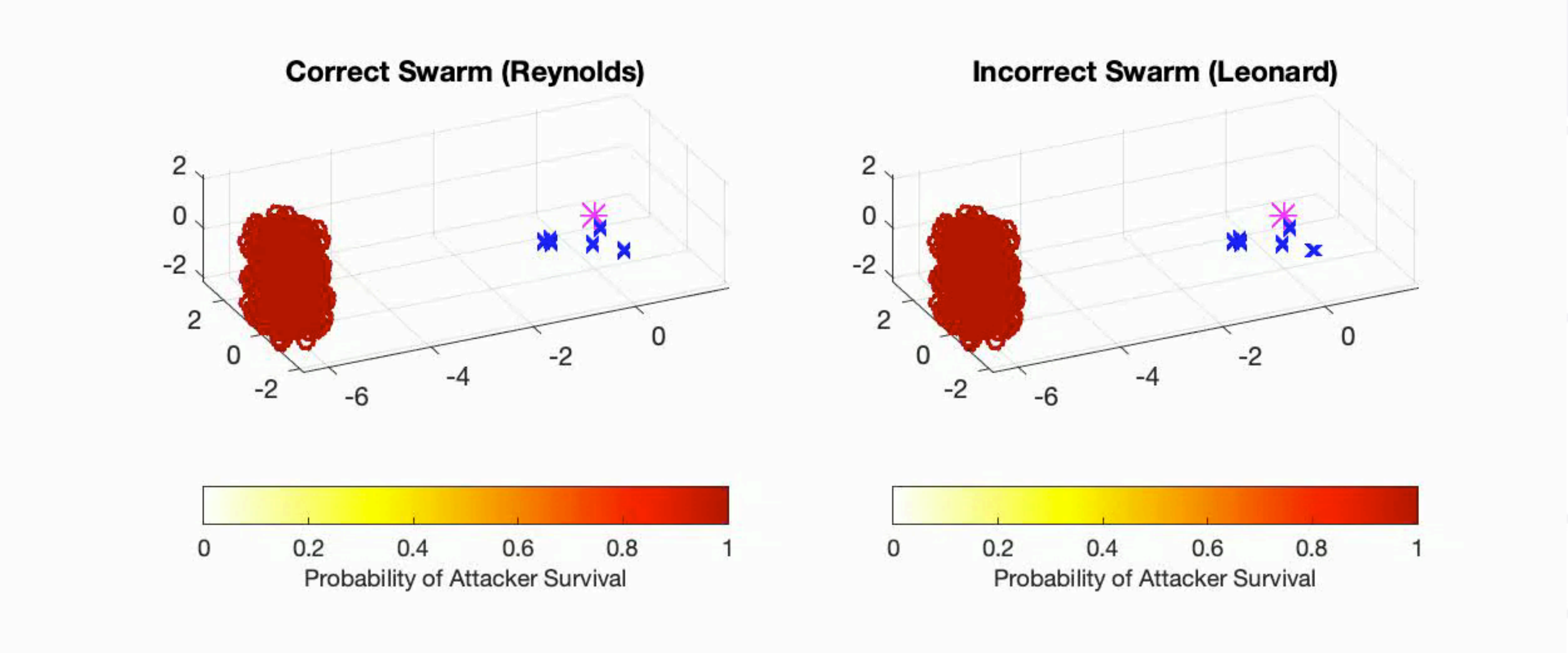


Swarm B

Swarm A: V. Cichella, I. Kaminer, C. Walton, N. Hovakimyan, 2018
Swarm B. N. Leonard and E. Fiorelli, 2004

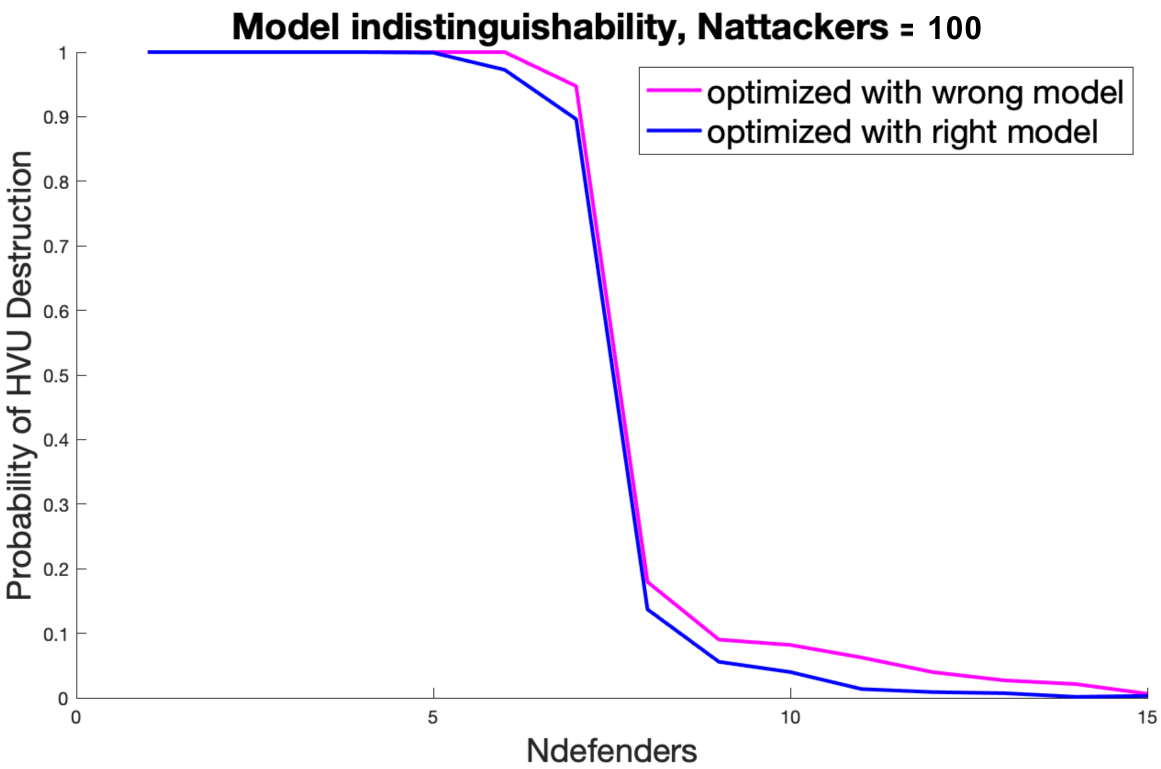
Black Box Robustness

- Robustness/Indistinguishability
  - **run estimation (partial observability)** using swarm strategy A (Reynolds)
  - obtain optimal defense for a swarm strategy A (Reynolds)
  - test on a swarm strategy B (Leonard)



Trade-off study

- Robustness/Indistinguishability



- **Robustness made possible using estimation – parallels to adaptive control**

Conclusions

- Rigorous theoretical and numerical framework to study adversarial swarming
  - i) nominal case
  - ii) in the presence of uncertainty

- Estimation
  - Partial Unobservabilty index
    - UKF is not always suitable
    - Optimization is a must
    - Trajectory and number of intruders matters
    - Time window matters
- Black Box Robustness

NAVAL POSTGRADUATE SCHOOL

Publications

- Kaminer I., A. Pascoal, E. Xargay, N. Hovakimyan, V. Cichella, and V. Dobrokhodov, "Time-Critical Cooperative Control of Autonomous Air Vehicles," Springer 2017, ISBN: 978-0-12-809946-9
- Chris Phelps, Qi Gong, Johannes O. Royset, Claire Walton and Isaac Kaminer, "Consistent Approximation of an Optimal Search Problem," Automatica 50.12 (2014): 2987-2997
- J.C. Foraker, J.O. Royset, and I. Kaminer, " Search-Trajectory Optimization: Part 1, Formulation and Theory," Journal of Optimization Theory and Applications, May 2016, Volume 169, pp 530-549, (DOI) 10.1007/s10957-015-0768-y
- J.C. Foraker, J.O. Royset, and I. Kaminer, " Search-Trajectory Optimization: Part 2, Algorithms and Computations," Journal of Optimization Theory and Applications, May 2016, 169(2), 550-567 (DOI) 10.1007/s10957-015-0770-4
- V. Cichella, I. Kaminer, C. Walton and N. Hovakimyan, "Optimal Motion Planning for Differentially Flat Systems Using Bernstein Approximation," IEEE Control Systems Letters, January 2018, pp. 181-186 Online ISSN: 2475-1456 Digital Object Identifier: 10.1109/LCSYS.2017.2778313
- Claire Walton, Panos Lambrianides, Isaac Kaminer, Johannes Royset, Qi Gong. "Optimal motion planning in rapid-fire combat situations with attacker uncertainty," NRL, April 2018, DOI:10.1002/nav.21790, pp 101 – 110
- Qi Gong, Wei Kang, Claire Walton, Isaac Kaminer, Hyeongjun Park, "Observability Analysis of an Adversarial Swarm's Cooperation Strategy," JGCD, Nov 2019, https://doi.org/10.2514/1.G004115
- Claire Walton, Isaac Kaminer, Qi Gong ,"Consistent Numerical Methods for State and Control Constrained Trajectory Optimization with Parameter Dependency," International Journal of Control, 2020, DOI: 10.1080/00207179.2020.1717633,https://doi.org/10.1080/00207179.2020.1717633 .
- V. Cichella, I. Kaminer, C. Walton and N. Hovakimyan, and A. Pascoal, "Optimal Multi-Vehicle Motion Planning using Bernstein Approximants," IEEE Transactions on Automatic Control, 2020. DOI: 10.1109/TAC.2020.2999329
- V. Cichella, I. Kaminer, C. Walton and N. Hovakimyan, and A. Pascoal, "Bernstein Approximation of ,Optimal Control Problems ," arXiv:1812.06132, Dec2018
- Sean Kragelund , Claire Walton, Isaac Kaminer, and Vladimir Dobrokhodov, "Generalized Optimal Control for Autonomous Mine Countermeasures Missions," IEEE Journal of Oceanic Engineering, 2020, DOI: 10.1109/JOE.2020.2998930
- Claire Walton, Isaac Kaminer, Qi Gong, Abe Clark and Theodoros Tsatsanifos, "Defense Against an Adversarial Swarm using Optimal Control with Parameter Uncertainty," **under review,** IEEE TCNS special issue Control of Very-large Scale Robotic (VLSR) Networks
- Theodoros Tsatsanifos, Abram H. Clark, Claire Walton, Isaac Kaminer, and Qi Gong, "Modeling Large-Scale Adversarial Swarm Engagements using Optimal Control," **accepted** 2021 IEEE CDC